**Math 35300: Sections 161 and 162. Linear algebra II** **Spring 2013**
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## Warm-up Questions 4

The following warm-up questions are intended to test your understanding of some of the basic definitions and constructions introduced in lecture. Your first step to answering these should be to go back to the lecture notes and read again the appropriate definition or construction. They will not be collected or graded.

**Question 1.** A map $f\colon V \longrightarrow W$ between vector spaces $V$ and $W$ over a field $\mathbb{F}$ is linear, if

(a) $f(ax+by) = af(x) + bf(y)$ for all $x, y \in V$, $a, b \in \mathbb{F}$.
(b) $f$ satisfies the eight axioms for a vector space.
(c) $f\colon V \longrightarrow W$ is bijective.

**Question 2.** By the kernel of a linear map $f\colon V \longrightarrow W$ one understands

(a) $\{w \in W \mid f(0) = w\}$
(b) $\{f(v) \mid v = 0\}$
(c) $\{v \in V \mid f(v) = 0\}$

**Question 3.** Which of the following statements are correct? If $f\colon V \longrightarrow W$ is a linear map, we have

(a) $f(0) = 0$.
(b) $f(-x) = -x$ for all $x \in V$.
(c) $f(av) = f(a) + f(v)$ for all $a \in \mathbb{F}$, $v \in V$.

**Question 4.** A linear map $f\colon V \longrightarrow W$ is called an isomorphism if

(a) there exists a linear map $g\colon W \longrightarrow V$ with $fg = \mathrm{Id}_W$ and $gf = \mathrm{Id}_V$.
(b) $V$ and $W$ are isomorphic.
(c) for each $n$-tuple $(v_1, \ldots, v_n)$ of vectors in $V$, the $n$-tuple $(f(v_1), \ldots, f(v_n))$ is a basis of $W$.

**Question 5.** By the rank $\mathrm{rk}(f)$ of a linear map $f\colon V \longrightarrow W$, one understands

(a) $\dim \operatorname{Ker} f$ (b) $\dim \operatorname{Im} f$ (c) $\dim W$

**Question 6.** $\begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} =$

(a) $\begin{bmatrix} 2 \\ 6 \end{bmatrix}$ (b) $\begin{bmatrix} 5 \\ -3 \end{bmatrix}$ (c) $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$

**Question 7.** The map $f\colon \mathbb{R}^2 \longrightarrow \mathbb{R}^2$, $(x, y) \longmapsto (x+y, x-y)$, is given by the following matrix ("The columns are the ..."):

(a) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

**Question 8.** Let $V$ and $W$ be vector spaces over a field $\mathbb{F}$ with bases $(v_1, v_2, v_3)$ and $(w_1, w_2, w_3)$, respectively, and let $f\colon V \longrightarrow W$ be the linear map with $f(v_i) = w_i$. Then the "associated" matrix is

(a) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

**Question 9.** A linear map $f\colon V \longrightarrow W$ is injective if and only if

(a) $f$ is surjective. (b) $\dim \operatorname{Ker} f = 0$. (c) $\operatorname{rk} f = 0$.

**Question 10.** Let $f\colon V \longrightarrow W$ be a surjective linear map and $\dim V = 5$, $\dim W = 3$. Then

(a) $\dim \operatorname{Ker} f \geq 3$.
(b) $\dim \operatorname{Ker} f$ is 0, 1, or 2 and each of these cases can arise.
(c) $\dim \operatorname{Ker} f = 2$.