Math 35300: Sections 161 and 162. Linear algebra II **Spring 2013**

John E. Harper
Purdue University

Warm-up Questions 7

The following warm-up questions are intended to test your understanding of some of the basic definitions and constructions introduced in lecture. Your first step to answering these should be to go back to the lecture notes and read again the appropriate definition or construction. They will not be collected or graded.

Question 1. The determinant is a map

(a) $\mathsf{M}(n \times n, \mathbb{F}) \longrightarrow \mathbb{F}$ given by the product of the diagonal elements.
(b) $\mathsf{M}(n \times n, \mathbb{F}) \longrightarrow \mathbb{F}$, which is linear in the rows, vanishes on matrices with less than maximal rank, and takes the value 1 on I.
(c) $\mathsf{M}(n \times n, \mathbb{F}) \longrightarrow \mathbb{F}^n$, which is given by a linear combination of rows, vanishes on matrices with less than maximal rank, and takes the value 1 on I.

Question 2. Let $A, A' \in \mathsf{M}(n \times n, \mathbb{F})$ and let A' be obtained from A by elementary row transformations. Which of the following statements are correct?

(a) $\det A = 0 \Longleftrightarrow \det A' = 0$
(b) $\det A = \det A'$.
(c) $\det A = c \det A'$ for some $c \in \mathbb{F}$, $c \neq 0$.

Question 3. Which of the following assertions is correct? For $A \in \mathsf{M}(n \times n, \mathbb{F})$ we have

(a) $\det A = 0 \Longrightarrow \operatorname{rk} A = 0$.
(b) $\det A = 0 \Longleftrightarrow \operatorname{rk} A \leq n - 1$.
(c) $\det A = 0 \Longrightarrow \operatorname{rk} A = n$.

Question 4. Which of the following statements holds for all $A, B, C \in \mathsf{M}(n \times n, \mathbb{F})$ and all $c \in \mathbb{F}$?

(a) $\det(A + B) = \det A + \det B$.
(b) $\det(cA) = c \det A$.
(c) $\det(ABC) = (\det A)(\det B)(\det C)$.

Question 5. Which of the formulas below is called "expansion of the determinant by minors on the i-th row"?

(a) $\det A = \sum_{i=1}^{n} (-1)^{i+j} a_{ij} \det A_{ij}$.
(b) $\det A = \sum_{j=1}^{n} (-1)^{i+j} a_{ij} \det A_{ji}$.
(c) $\det A = \sum_{j=1}^{n} (-1)^{i+j} a_{ij} \det A_{ij}$.

Question 6. $\det \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & -1 \\ 0 & 1 & 1 \end{bmatrix} =$

(a) 2 (b) 4 (c) 6

Question 7. Let $I \in \mathsf{M}(n \times n, \mathbb{F})$ be the unit matrix. Then the transposed matrix $I^t =$

(a) $\begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix}$ (b) $\begin{bmatrix} & & 1 \\ & \iddots & \\ 1 & & \end{bmatrix}$ (c) $\begin{bmatrix} t & & \\ & \ddots & \\ & & t \end{bmatrix}$

Question 8. $\det \begin{bmatrix} c & c & c \\ c & c & c \\ c & c & c \end{bmatrix} =$

(a) 0 (b) c (c) c^3

Question 9. $\det \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix} =$

(a) $\cos 2\varphi$ (b) 0 (c) 1

Question 10. Which of the following is (or are) false?

(a) $\det A = 1 \Longrightarrow A = I$.
(b) $\det A = 1 \Longrightarrow A$ is injective as a map $\mathbb{F}^n \longrightarrow \mathbb{F}^n$.
(c) $\det A = 1 \Longrightarrow A$ is surjective as a map $\mathbb{F}^n \longrightarrow \mathbb{F}^n$.