WORKSHOP ON FUNCTOR CALCULUS (OSU)
March 16–17, 2019

The first talk on Saturday (March 16) will begin at 8:30am, and the workshop will end on Sunday (March 17) at 1pm.

Talks in EA 160 and EA 170
Building EA is next to the Math Tower at:
209 W 18th Ave, Columbus, OH 43210

Parking Suggestions:
(i) Tuttle Garage (OSU campus): about a 4-minute walk to Building EA
2050 Tuttle Park Place, Columbus, OH 43210

(ii) Lane Avenue Garage (OSU campus): about a 10-minute walk to Building EA
2105 Neil Avenue, Columbus, OH 43210

Organizers: Ernie Fontes, John E. Harper, Christina Osborne, Nath Rao.

We gratefully acknowledge funding from (i) the Math Research Institute (MRI) under the "Ohio State Department of Mathematics Thematic Year on Topology" and (ii) the NSF-RTG #1547357 "Algebraic Topology and its Applications".

Titles and Abstracts

1. *Invited Talks: (two 1-hour talks per speaker)*

Tom Goodwillie (Brown). Title: *Origins of functor calculus*

Abstract: One thing that can get lost when mathematical ideas are published is clues about where the ideas came from. I will discuss the origins of the "manifold" flavor of functor calculus in early work of mine, as well as the way that the "homotopy" flavor of functor calculus originally grew out of ideas about manifolds.

Tom Goodwillie (Brown). Title: *Graphs, manifolds, and orthogonal calculus*

Abstract: This is about long-term work in progress. The discrepancy between homotopy theory and the world of manifolds is accounted for by (L-theory and) pseudoisotopy theory. The most prominent and accessible part of pseudoisotopy theory, the "stable" part of it, is related to algebraic K-theory and is to some extent detected by "trace" tools, which are related to the space of maps $S^1 \to M$ from a circle into a manifold. Looking beyond the stable part, one can try to develop analogous tools to detect more of it. These will be related to maps $\Gamma \to M$ where Γ is a graph more complicated than the circle.

Michael Weiss (Muenster). Title: *Spaces of smooth embeddings, configuration categories and applications to Pontryagin classes*

Abstract: In manifold calculus, it can be fruitful to put calculus notions aside and to think more in terms of localizations. Multiple disjunction theory makes it possible. This observation should lead on to the concept of configuration category. Returning to calculus notions, we can then understand better how in the Taylor tower for spaces of smooth embeddings, the linear algebra (which we see abundantly in the first layer) is tied to the homotopical properties of configuration spaces of the two manifolds (which we see abundantly in the higher layers). Application: construction of families of smooth embeddings with wild derivatives. Some conclusions regarding rational Pontryagin classes of fiber bundles with m-dimensional euclidean space as fiber.

Michael Weiss (Muenster). Title: *Orthogonal calculus first derivatives and applications to exotic spheres*

Abstract: Orthogonal calculus, especially the first derivatives aspect, has some precocious beginnings in the paper by Frank Adams "Vector fields on spheres" and related earlier papers by Ioan James. It is not a great exaggeration to say that Adams and James relied on the first OC Taylor aproximation of the functor taking V to $\mathrm{BO}(V)$, where V is a f.d. real vector space with inner product. Taking that point of view, we may want to investigate similar functors such as the one taking V to $\mathrm{BTOP}(V)$ or to $\mathrm{TOP}(V)/O(V)$, concentrating again on first derivative aspects. By means of smoothing theory, these functors are closely related to the geometry of smooth homotopy spheres. Rather than zooming in on the question which is the closest analogue to the vector fields on spheres question, I plan to talk about a less important but apparently harder problem which is as follows. Let M be a smooth homotopy sphere of dimension 7, equipped with a base point. Does the map "evaluation at the base point" from $\mathrm{Diff}(M)$ to M admit a section? Clearly it does if M is the standard 7-sphere.

2. *Invited Talks: (one 1-hour talk per speaker)*

Michael Ching (Amherst). Title: *Pro-operads and the classification of Taylor towers for spectrum-valued functors*

Abstract: The Goodwillie derivatives of the identity functor on a (pointed compactly-generated) infinity-category C possess an operad structure which reduces to a spectral version of the Lie algebra for the infinity-category of pointed spaces. In this talk I will describe a refinement of this structure to that of a pro-operad. Much of the talk will be concerned with proposing and justifying a suitable definition of pro-operad, and of module over a pro-operad of spectra. I will then explain how the derivatives of a spectrum-valued functor on C form a module over the corresponding pro-operad, and that this module structure completely classifies the Taylor tower of such functors.

Brenda Johnson (Union). Title: *Abelian functor calculus*

Abstract: This talk will provide an introduction to abelian functor calculus, a calculus for functors of abelian categories with roots in work of Dold and Puppe, and Eilenberg and Mac Lane. After describing polynomial functors and Taylor towers in this context, I will discuss how this calculus is related to the Cartesian differential categories of Blute, Cockett, and Seely, and how it provides a template for building new functor calculi.

John Klein (Wayne State). Title: *Poincaré calculus*

Abstract: The proof of convergence of the embedding calculus tower in codimensions at least three makes use of disjunction results for spaces of Poincaré embeddings. This motivates the question of whether there is a version of manifold calculus in which manifolds are replaced by Poincaré duality spaces. In my talk, I hope to describe what such a calculus might look like.

Ayelet Lindenstrauss (Indiana). Title: *The Goodwillie Taylor tower of algebraic K-theory*

Abstract: I will discuss my calculation with Randy McCarthy of the Goodwillie Taylor tower of algebraic K-theory, and more precisely: of the functor that sends a simplicial set X to the reduced (over the K-theory of R) algebraic K theory of the simplicial ring which is the square zero extension of a ring R by the free simplicial module on X with coefficients in an R-module M.

Algebraic K-theory of rings is very hard to calculate. The Dennis trace map sends it to the much easier-to-calculate invariant Hochschild homology, but unfortunately that approximation is not as good as one would hope. Before the days of "good" (strictly unital and associative) ring spectra, Tom Goodwillie conjectured that if one mimicked the construction of Hochschild homology for ring spectra, one would get an invariant (topological Hochschild homology) which would agree with stable K-theory, which is also the derivative in the sense of Goodwillie calculus of the algebraic K-theory functor of spaces that was described above. Marcel Boekstedt constructed this new invariant, and Bjorn Dundas and Randy McCarthy showed that indeed it was stable K-theory. I will talk about their work, how it inspired the construction Randy McCarthy and I did for the full Goodwillie Taylor tower, and how we used it to prove that our construction really was the correct tower.

3. *Contributed Talks: (14 contributed talks of 20-minutes each in two parallel sessions)*

Jacobson Blomquist (Binghamton). Title: *Iterated desuspension and delooping of structured ring spectra*

Abstract: The completion of a space with respect to ordinary homology (with coefficients in a ring), originally due to Sullivan and Bousfield-Kan, throws away all information other than the part of the space that homology sees. Bousfield-Kan showed that a one connected space will be homotopy equivalent to its homology completion. Hopkins, and later Bousfield, studied the completion of a space with respect to finite loop-suspension, and Carlsson and Arone-Kankaanrinta studied the case of stabilization, and in both cases a similar result exists.

These constructions can be done in the context of algebras over a reduced operad $\mathcal{O}$ in spectra. In this setting topological Quillen (TQ) homology, the appropriate analog of homology, agrees with stabilization. Ching-Harper proved, using combinatorial techniques, that 0-connected $\mathcal{O}$-algebras are equivalent to their TQ completion, and in fact that this completion extends to an equivalence of homotopy categories between 0-connected $\mathcal{O}$-algebras and coalgebras over a TQ-comonad. In this talk I will discuss the case of completion with respect to finite loop-suspension for $\mathcal{O}$-algebras and outline the proof that iterated suspension gives an equivalence of homotopy theories between 0-connected $\mathcal{O}$-algebras and r-connected $\Sigma^r\Omega^r$-coalgebras, using an analysis of cubical diagrams and the higher (dual) Blakers-Massey theorems due to Ching-Harper, which also gives a new proof of their result for TQ homology. Time permitting I will also discuss the dual case, that iterated loops gives an equivalence between homotopy theories between r-connected $\mathcal{O}$-algebras and 0-connected $\Omega^r\Sigma^r$-algebras.

Duncan Clark (OSU). Title: *Understanding higher excision: from stabilization to better approximations*

Abstract: In this expository talk, beginning with the Freudenthal suspension theorem, we motivate an intuitive understanding of Goodwillie's Taylor tower and the construction of n-excisive approximations to the identity functor on based spaces via the higher Blakers-Massey theorems. Though fundamental, the aim of this talk is to suggest how one might "discover for oneself" some of the core concepts and definitions that underlie the Taylor tower in functor calculus.

Kyle Ferendo (Brown). Title: *n-Excisive infinity-categories and Tate coalgebras*

Abstract: This expository talk will summarize some of the main results of a paper by Gijs Heuts entitled "Goodwillie approximations to higher categories". This paper introduces the notion of n-excisive infinity-categories, generalizing the notion of a stable infinity-category, which is synonymous with a 1-excisive infinity-category. For each n, the category of presentable n-excisive infinity-categories forms a reflective subcategory of the category of presentable infinity-categories. We describe two models for the n-excisive approximation of an infinity-category. The first is a straightforward but abstract construction; the second, which uses Tate coalgebras in stable operads, while more challenging to describe, is more conceptually compelling.

Keely Grossnickle (Kansas State). Title: *Overlapping discs and spaces of immersions*

Abstract: Configuration spaces of non-k-overlapping discs have been studied as a bimodule but they also form a filtered operad. I study the induced structure in homology. The bimodule structure naturally appears in the study of Spaces of non-k-equal Immersions $D^m \to D^n$ by means of the Goodwillie-Weiss functor calculus and can also be used to obtain the $(m+1)$-st delooping of the corresponding Taylor tower. We believe that the structure of a filtered operad on overlapping discs could be used to understand the associated filtered B_{m+1}-algebra structure and the Browder operator on the Space of Disc Immersions filtered by the degree of the overlap.

Philip Hackney (Louisiana). Title: *A graph category for higher modular operads*

Abstract: Modular operads (introduced by Getzler and Kapranov) are a generalization of cyclic operads which allow one to model algebras which come equipped with traces. Recent applications have made clear the need for a homotopy coherent version of this notion. In this talk we present a new category of undirected graphs which is suitable for providing a foundation for such a theory.

Jens Kjaer (Notre Dame). Title: *Unstable v_1-periodic homotopy groups through Goodwillie calculus*

Abstract: It is a classical result that the rational homotopy groups, $\pi_*(X) \otimes \mathbb{Q}$, as a Lie-algebra can be computed in terms of indecomposable elements of the rational cochains on X. This can also be recovered from applying Goodwillie calculus to rational homotopy theory. A different simplification of the homotopy theory, is v_h-periodic homotopy theory. For $h = 1$ we are able to compute the K-theory based v_1-periodic Goodwillie spectral sequence in terms of derived indecomposables. This allows us to compute $v_1^{-1}\pi_* SU(d)$ in a very different way from the original computation by Davis.

Robin Koytcheff (Louisiana). Title: *Operad actions on the Taylor tower for the space of knots*

Abstract: I will discuss the Taylor tower for the space of long 1-dimensional knots in Euclidean space. When the codimension is at least 3, this tower converges to the space of knots, while in the classical case of codimension 2, all real-valued finite-type invariants factor through it. With Budney, Conant, and Sinha, we constructed a homotopy-commutative multiplication on each stage of the tower compatible with stacking long knots via the evaluation map. This helped us provide evidence for a conjecture that the tower is a universal abelian-group-valued finite-type invariant. In ongoing work with Zhang, we will promote this multiplication to an E_2 operad action, and in ongoing work with Budney, we will promote it to an action of the splicing operad.

Cynthia Lester (Oregon). Title: *The homotopical canonical Grothendieck topology*

Abstract: I will explain that every simplicial model category comes equipped with a canonical Grothendieck topology, called the canonical homotopical topology. This topology is defined using homotopy colimits and is a generalization of the classical non-homotopical canonical topology from sheaf theory. Additionally, I will discuss some open questions on how this Grothendieck topology interacts with the homotopical information in the model category and showcase some examples.

Apurva Nakade (Johns Hopkins). Title: *Manifold calculus and h-principle*

Abstract: In this talk, I'll talk about applying manifold calculus to study the Lagrangian embeddings in a symplectic manifold, proving that the analytic approximation of the Lagrangian embeddings functor is the totally real embeddings functor using Gromov's h-principle.

Peter Patzt (Purdue). Title: *Polynomial functors and representation stability*

Abstract: A sequence of groups or spaces exhibit homological stability if their homology stabilizes. Representation stability is an equivariant form of homological stability, with group actions of another sequence of groups. Over the symmetric groups, representation stability is equivalent to being a polynomial functor. In joint work with Jeremy Miller and Jenny Wilson, we show that over the general linear groups and the symplectic groups of PIDs a polynomial functor is representation stable. The converse is not true. This result implies representation stability of the second homology of Torelli groups.

Yuri Sulyma (UT Austin). Title: *The Dundas-Goodwillie-McCarthy theorem and trace methods*

Abstract: One of the major applications of functor calculus is the Dundas-Goodwillie-McCarthy theorem, which states that the difference between algebraic K-theory and topological cyclic homology (as measured by the fiber of the cyclotomic trace) is "locally constant". This enables the calculation of K-theory by "trace methods", which has led to spectacular advances in our understanding of K-theory, for example the work of Hesselholt and Madsen. In this expository talk, I will sketch the proof of the DGM theorem, mostly following the note by Raskin, and sample some applications.

Paul Tsopmene (Regina). Title: *Cosimplicial models for manifold calculus*

Abstract: Manifold calculus is a tool developed by Goodwillie and Weiss which enables to approximate a contravariant functor, F, from the category of m-manifolds to the category of spaces (or alike), by its "Taylor approximation", $T_\infty F$. I will present a fairly explicit and computable cosimplicial model of $T_\infty F(M)$ constructed out of a simplicial model of the manifold M (i.e. out of a simplicial set whose geometric realization is M). This cosimplicial model in degree p is then equivalent to the evaluation of F on a disjoint union of as many m-disks as p-simplices in the simplicial model of M.

As an example, we apply this construction to the functor $F(M) = \mathrm{Emb}(M; W)$ of smooth embeddings in a given manifold W; in that case our cosimplicial model in degree p is then just the (framed) configuration space of all the p-simplices of M in W. When $\dim(W) > \dim(M)+2$, a theorem of Goodwillie-Klein-Weiss implies that our explicit cosimplicial space is a model for $\mathrm{Emb}(M; W)$. This generalizes Sinha's cosimplicial model for the space of long knots which was for the special case when M is the real line.

David White (Denison). Title: *Model categories and functor calculus*

Abstract: I will present a new model category structure that encodes the homotopy theory of (small) homotopy functors, from a combinatorial model category to simplicial sets. The fibrant objects are the homotopy functors, i.e. functors that preserve weak equivalences. Next, I will explain how the homotopy theory of homotopy functors is homotopy invariant, i.e. a Quillen equivalence on domain categories induces a Quillen equivalence on homotopy functor categories. This result has numerous applications, including spaces, spectra, chain complexes, simplicial presheaves, motivic spectra, infinity categories, and infinity operads. If there is time, I will connect this model category to ongoing work in functor calculus. This is joint work with Boris Chorny.

Sarah Yeakel (Maryland). Title: *Chain rules and operads in abelian functor calculus*

Abstract: The abelian functor calculus of Johnson and McCarthy associates to a functor of abelian categories a sequence of polynomialäpproximations analogous to a Taylor series. Work of Bauer, Johnson, Osborne, Riehl, and Tebbe shows that in this setting, a directional derivative can be defined which yields a higher order chain rule. In ongoing work with Bauer and Johnson, we have found another chain rule, this time for the higher order differential. Using this chain rule, we define an operad structure on the derivatives of monads of R-modules, and more generally, find a monoidal structure for the derivatives of functors of abelian categories. We expect this to lead to classifications of homogeneous and polynomial functors of abelian categories.

WORKSHOP ON FUNCTOR CALCULUS (OSU)
March 16–17, 2019

The first talk on Saturday (March 16) will begin at 8:30am, and the workshop will end on Sunday (March 17) at 1pm.

Talks in EA 160 and EA 170
Building EA is next to the Math Tower at:
209 W 18th Ave, Columbus, OH 43210

Parking Suggestions:
(i) Tuttle Garage (OSU campus): about a 4-minute walk to Building EA
2050 Tuttle Park Place, Columbus, OH 43210

(ii) Lane Avenue Garage (OSU campus): about a 10-minute walk to Building EA
2105 Neil Avenue, Columbus, OH 43210

Organizers: Ernie Fontes, John E. Harper, Christina Osborne, Nath Rao.

We gratefully acknowledge funding from (i) the Math Research Institute (MRI) under the "Ohio State Department of Mathematics Thematic Year on Topology" and (ii) the NSF-RTG #1547357 "Algebraic Topology and its Applications".

Schedule of Talks

Saturday: Invited Talks (1-hour each in EA 160)

7:30am: **Coffee, Bagels, and Fruit.** (EA lobby)

8:30am: **Brenda Johnson (Union).** *Abelian functor calculus*

9:50am: **Michael Weiss (Muenster).** *Spaces of smooth embeddings, configuration categories and applications to Pontryagin classes*

11:10am: **Tom Goodwillie (Brown).** *Origins of functor calculus*

12:10pm: **Lunch Break.**

2:00pm: **Ayelet Lindenstrauss (Indiana).** *The Goodwillie Taylor tower of algebraic K-theory*

Saturday: Contributed Talks (20-minutes each in two parallel sessions in EA 160 and EA 170)

3:10pm: **Duncan Clark (OSU).** (EA 160): *Understanding higher excision: from stabilization to better approximations.*

3:10pm: **Keely Grossnickle (Kansas State).** (EA 170): *Overlapping discs and spaces of immersions*

3:40pm: **Kyle Ferendo (Brown).** (EA 160): *n-Excisive infinity-categories and Tate coalgebras*

3:40pm: **Peter Patzt (Purdue).** (EA 170): *Polynomial functors and representation stability*

4:10pm: **Sarah Yeakel (Maryland).** (EA 160): *Chain rules and operads in abelian functor calculus*

4:10pm: **Robin Koytcheff (Louisiana).** (EA 170): *Operad actions on the Taylor tower for the space of knots*

4:30pm: **Coffee Break: with Cookies and Scones.** (EA lobby)

5:00pm: **Paul Tsopmene (Regina).** (EA 160): *Cosimplicial models for manifold calculus*

5:00pm: **Cynthia Lester (Oregon).** (EA 170): *The homotopical canonical Grothendieck topology*

5:30pm: **Jacobson Blomquist (Binghamton).** (EA 160): *Iterated desuspension and delooping of structured ring spectra*

5:30pm: **Philip Hackney (Louisiana).** (EA 170): *A graph category for higher modular operads*

6:00pm: **Yuri Sulyma (UT Austin).** (EA 160): *The Dundas-Goodwillie-McCarthy theorem and trace methods*

6:00pm: **Jens Kjaer (Notre Dame).** (EA 170): *Unstable v_1-periodic homotopy groups through Goodwillie calculus*

6:30pm: **Apurva Nakade (Johns Hopkins).** (EA 160): *Manifold calculus and h-principle*

6:30pm: **David White (Denison).** (EA 170): *Model categories and functor calculus*

Schedule of Talks (cont.)

Sunday: Invited Talks (1-hour each in EA 160)

7:00am: **Coffee, Bagels, Pastries, and Fruit.** (EA lobby)

8:00am: **Michael Ching (Amherst).** *Pro-operads and the classification of Taylor towers for spectrum-valued functors*

9:20am: **Michael Weiss (Muenster).** *Orthogonal calculus first derivatives and applications to exotic spheres*

10:40am: **Tom Goodwillie (Brown).** *Graphs, manifolds, and orthogonal calculus*

12:00pm: **John Klein (Wayne State).** *Poincaré calculus*

1:00pm: **End of Workshop**