Vilas Winstein - Project Proposal - Spring 2017 URS

The Riemann zeta function is an important function in Number Theory, and is central to the Riemann hypothesis, which is considered one of the greatest unsolved problems in mathematics. My project is to implement a new method for computing the Riemann zeta function with very high accuracy.

The Riemann zeta function is defined by an infinite series and so cannot be numerically evaluated exactly for all input values. Current methods for computing the zeta function include the Riemann-Siegel formula which is fast and gives a good general approximation but has limited accuracy. There are also methods (such as the Euler-Maclaurin formula) which are provably very accurate, but are very slow for large input values. The method I will be implementing is a new formula that is fast for large input values, doesn't suffer from limited accuracy, and generalizes to a class of zeta functions called Dirichlet L-functions[1] to powerful modulus.

The new formula is significant because it gives a new accurate method to numerically approximate zeroes of the Riemann zeta function (input values for which the function evaluates to zero) which are the subject of an active research area. The usual strategy to do that is to use the Intermediate Value Theorem, simply locating zeros by detecting a sign change (from positive to negative or vice versa) and zooming in via a bisection algorithm. Therefore, the main effort is in point-wise evaluations of the zeta function. If the accuracy of the computed output of the zeta function is higher, the exact location of these zeroes can also be found with higher accuracy.

The new formula for the Riemann zeta function was discovered by Dr. Ghaith Hiary, and while the theoretical details of the formula are available in [3], much work is needed to ensure that the formula is practical. The paper that Dr. Hiary published emphasizes computational complexity aspects, and does not immediately enable a practical implementation on a computer system. Therefore, the main purpose of this project is to determine how exactly to implement and optimize the new formula to run as quickly as possible on various computer systems, using an array of techniques, possibly including multiprocessing for enhanced performance on computer systems with multiple processing cores. Another goal is to examine how to generalize this computation to some Dirichlet L-Functions.

I'll be working alongside Dr. Hiary to implement this formula in the C programming language. Then I'll do timing experiments and modify the code to make it run as fast as possible. I have already done some preliminary work on implementing the formula, such as setting up a library of functions that perform operations on complex numbers using the GNU MPFR library for arbitrary precision arithmetic, and implementing the Euler-Maclaurin formula in arbitrary precision.

Once the project is completed, I plan on making a project poster and presenting at various poster presentations such as the Fall Student Poster Forum and the Denman Undergraduate Research Forum. As well as that, I plan to make a web page summarizing the project and providing some sample high precision calculations generated by the formula. I will also be making the source code freely available on the web page and on GitHub in case the formula and its practical implementation are of interest to other researchers in the field.

Bibliography

[1] Apostol, Tom M. Introduction to Analytic Number Theory. New York: Springer-Verlag, 1976. Print.

[2] Edwards, Harold M. Riemann's Zeta Function. New York: Academic, 1974. Print.

[3] Hiary, G. A. (29 Aug 2015). *An alternative to Riemann-Siegel type formulas.*