Quiz 2

Instructions: This quiz is worth a total of 10 points. You may use any notes or books but you must work individually. The only computation aid which you may use is MATLAB, unless otherwise indicated. Make sure to write clearly and justify your answers.

(1) (2 pts.) Let \( A = \begin{pmatrix} 1 & 2 & 5 \\ -4 & -7 & -12 \\ 6 & 7 & -3 \end{pmatrix} \) and \( B = \begin{pmatrix} 1 & 2 & 5 \\ 0 & 1 & 8 \\ 0 & 0 & 1 \end{pmatrix} \)

Find four elementary matrices \( E_1, E_2, E_3 \) and \( E_4 \) such that \( E_4 E_3 E_2 E_1 A = B \)

(2) (3 pts.) Find the inverse of \( A = \begin{pmatrix} 1 & 2 & 5 \\ -4 & -7 & -12 \\ 6 & 7 & -3 \end{pmatrix} \). DO NOT USE MATLAB

(3) (3 pts.) Find the \( LU \)-decomposition of \( A = \begin{pmatrix} 2 & 4 & 5 \\ 6 & 9 & 23 \\ -8 & 8 & -90 \end{pmatrix} \). DO NOT USE MATLAB

(4) (2 pts.) Let \( A = \begin{pmatrix} 5 & 4 \\ 5 & -3 \end{pmatrix} \)

(a.) Compute \( A - \lambda I_2 \) where \( I_2 \) is the \( 2 \times 2 \) identity matrix and \( \lambda \) is an unknown.

(b.) Find \( d(\lambda) = \det(A - \lambda I) \). Note that this will be a polynomial in \( \lambda \).

(c.) Find the roots of \( d(\lambda) \).

(d.) If \( \lambda_1 \) and \( \lambda_2 \) are the roots found in (c.), compute:
   (i.) \( A - \lambda_1 I_2 \)
   (ii.) \( A - \lambda_2 I_2 \)
   (iii.) \( (A - \lambda_1 I_2)(A - \lambda_2 I_2) \)