Math 117

Final Exam Review

Spring Quarter 2006

Structure of Exam

1-2 Max/Min problems (stories and/or given function)
1-2 Given a situation, what does a given integral or derivative represent (or vice versa: write down an integral or derivative to represent a desired quantity).
3-4 Applications of Integrals of what was covered in the course: set up a definite integral to represent a physical quantity, such as area between curves, center of mass, length of curve, average value of a function, hydrostatic force, volume by slicing, volume of revolution
0-1 Concept question about an integral (e.g., p. 383 #3, 19-20)
0-1 Given f'' and initial conditions, find info about f
0-1 Approximate an integral using Simpson's Rule (May be story problem with data)
3-4 Compute an antiderivative (e.g., by substitution or table)
0-1 Linear Approximations and Differentials
0-1 Curve Sketching Problem

Text Review Problems

Any problem you are responsible for (from syllabus) and:
p. 336: 1-8, 13-15, 37-40, 44, 49-54
p. 434: 2, 3, 7, 9-30, 63-66
p. 494: 1-2, 4, 6ab, 7ac, 8abc, 9, 11, 12, 16-17, 22, 24, 26

Other Review Problems and Old Exams

1. Sketch the graph of $f(x) = x^4 - 2x^3$. Be sure to include (and label) any x-intercepts, relative extrema, and points of inflection (found using calculus techniques). When sketching, be careful to include the shape of the graph with respect to increasing/decreasing and its concavity. YOU MAY NOT USE YOUR GRAPHING CALCULATOR EXCEPT AS A CHECKING DEVICE.
2. Given $f(x) = 2x^3 - 3x^2 - 12x + 5$, find the absolute maximum and absolute minimum values of f on [-2,4].
3. A rectangular box-shaped house is to have a square floor. Three times as much heat per square foot enters through the roof as through the walls. What dimensions should the house be if it is to enclose a volume of 12000 cubic feet and minimize heat entry? (Assume no heat enters through the floor and assume that heat is proportional to area with proportionality constant 1).

4. Find the absolute maximum and absolute minimum values of f on the given interval:

   a. $f(x) = \dfrac{x}{x^2+1}$ on [0,2]

   b. $f(x) = \sin x + \cos x$ on [0,π/3] (if in radian mode. In degree mode, it would be [0, 60])

   c. $f(x) = x - 3\ln x$ on [1,4]

5. Assuming f and f' are defined for all values of x, what would you look for on the graph of f' to see on what intervals f is increasing or decreasing? At what values of x would you look at on the graph of f'' to see where f had a maximum or minimum? How would you tell from the graph of f' if those values of x gave a max, min, or neither for f?
6. A man launches his boat from point A on a bank of a straight river that is 3 miles wide and wants to reach point B, 8 miles downstream as quickly as possible. If he can row at 6 mph and run at 8 mph, where should he land on the opposite bank to reach point B as quickly as possible? Assume that the speed of the water is negligible.
7. A cone-shaped paper drinking cup is to be made to hold 27 cubic centimeters of water. Find the height and radius of the cup that will use the smallest amount of paper. (Fun Facts: Volume of a cone = $\frac{1}{3}\pi r^2 h$ and the surface area of a cone = $\pi r\sqrt{r^2+h^2}$)
8. A painting in an art gallery has height h and is hung so that its lower edge is a distance d above the eye of an observer. How far from the wall should the observer stand to get the best view? (That is, where should the observer stand so as to maximize the angle subtended at his eye by the painting?
9. A section of roller coaster is in the shape of $y = \frac{1}{5}x^5 - \frac{5}{3}x^3 - 36x + 250$ for x in [-4, 4]. Assume riders go left to right (as x increases). Assume you do not have access to a picture of the coaster.

   (**HELPFUL NOTES**: The derivative turns out to be quadratic in form. Also, the x values are *restricted* on a closed interval!)

   a. (4 points) What information could you find to be able to tell if the coaster going up or down at x=2?

   b. (4 points) What information could you find to be able to tell if the coaster is speeding up or slowing down at x=2?

   c. (6 points) Find the highest and lowest points on the coaster for x in [-4,4].

   d. (6 points) Find where the coaster is steepest (regardless of direction) for x in [-4, 4].
10. Given a graph of $y = f(x)$ and that $f''(x)$ is zero only at x=3,

a. (6 points) Know how to sketch the graph of $f'(x)$. Be sure to include any x-intercepts of that graph.
b. (6 points) Know how to sketch the graph of $f''(x)$. Be sure to include any x-intercepts of that graph.

11.. Given $f(x)=3x^4-4x^3-12x^2+5$, follow the steps to ward sketching a graph of the function that includes its x-intercepts, relative extrema, and inflection points. (**FUN FACTS:** $f(x)=0$ only when x=-1, x=.612, and x = 2.72. Also, $\frac{2\pm\sqrt{28}}{6}=1.21$ and -.549.)

a. (2 points) Find any x-intercepts of f. You need not find where f is above or below the x-axis.

b. (6 points) Find any critical values of f (along with relative max and mins) and for what x-values f is increasing or decreasing.

c. (6 points) Find any points of inflection of f and for what x-values f is concave up or concave down.

d. (6 points) Using only the information you found in parts (a), (b), and (c), sketch the graph of f. Be sure to include any x-intercepts, relative extrema, and points of inflection. (NOTE: Your graph should reflect your work and results above. Your graphing calculator can only be used as a checking device).

12. A graphing artist is designing a rectangular poster, which is to have margins of 2 inches at the top and along each side, and a 3 inch margin at the bottom. In order to save expenses, she wants the total area of the poster to be as small as possible, but the printed area (the part inside the margins) has to be 180 square inches. What dimensions of the poster will minimize the total area? Be sure to include evidence that your dimensions do give a minimal area.

13. a. A football punt follows the path $y=\frac{1}{15}x(60-x)$ yards. You have previously (here or in other courses) found how far the punt went horizontally (set y=0 and solve for x) and how high it went (maximize the function). Now find how far the football actually traveled.

b. Do the same as in (a), but for a baseball thrown from center field that follows the path $y=\frac{1}{300}x(100-x)$ yards. Why would the baseball player want a small arc length while the football player in part (a) would want a large arc length?

14. In a certain city, the temperature (in Fahrenheit) t hours after 9 AM was modeled by the function:

$f(t)=50+14\sin(\frac{\pi t}{12})$. Find the average temperature during the period from 9AM to 7PM. Do the same for between 11AM and 2PM.

15. Use integration to derive the formulas for area of a circle of radius r and volume of a sphere of radius r.

16. Suppose that rainfall begins at noon and the rate of rainfall t hours later is given by $\frac{1}{12}(t+6)$ inches per hour. How many inches of rain fell during the first 5 hours? Between the 3:45PM and 5PM?
17. Use Simpson's Rule to approximate ln(5) (Hint: Do you know a function whose integral is ln(x)?).
18. A 30-inch baseball bat can be represented approximately by an object with density function $\delta(x) = 512(\frac{1}{46}+\frac{x}{690})^2$ ounces per square inch. This function takes into account that a bat is like an elongated cone. Find the mass of the bat. Find the center of mass of the bat (i.e., the "sweet spot").
19. (The notion of "curvature"- **NOT for the exam**, but an important topic, particularly for Architecture majors!): Often, we want to find out how "curvy" a curve is at a point or over an interval. That is, the more "turn" the curve has, the bigger its curvature, while the closer the curve is to being like a line, the less curvature.

The study of curvature *over an interval* is related to an application of integration we've studied: the length of a curve. One way to quantify the curvature of a function f(x) over an interval is to compare the arc length of f(x) on the interval (call it L_1) to the length of the secant line connecting the end points of the graph (call it L_2). The ratio L_2/L_1 must always be 1 or less because the shortest distance between 2 points is the length of the segment connecting them (thus L_2 must be less than or equal to L_1). If the ratio is close to one, then the graph of f is relatively "straight" over the interval (i.e., the lengths are close- the graph doesn't deviate much from a straight line). However, the smaller the ratio is, the more "curvy" the graph is over the interval (i.e., the arc length is much bigger- suggesting that the graph either big or many turns in the interval). For example, find the ratio for the graph of $f(x)=x^2$ on [-1, 3] and then on [8, 10].

How do we tell about curvature *at a point*? Here, we are asking by how much is the *direction* of a graph is changing as x changes (as opposed to when we studied derivatives and how fgst the y-values were changing as x changes). We would expect that a line would have zero curvature, since it never changes direction. We would expect a circle to have constant curvature, since it you could "drive" around it without altering the orientation of your "steering wheel". Other graphs would have varying curvature at each point: on "turns", there would be more curvature than on "straight-a-ways". It turns out that mathematicians define curvature to be the "rate of the change of the *angle formed by the tangent line and the x-axis* (technically the unit tangent vector) with respect to *the arc length of the curve*". That is, if the tangent line changes slope a lot over a short distance, the curvature is great. If that slope doesn't change a lot (close to linear behavior), there is less curvature. For a curve defined by a function f(x), it turns out the formula for curvature at a point a is given by:

$$\kappa = \frac{f''(a)}{(1+[f'(a)]^2)^{\frac{3}{2}}}$$

This makes some sense if you consider that the second derivative (the numerator) measures concavity (the more acceleration on a curve, the more turning), while the denominator is "basically" the arc length formula (the integral of the square root of the length expression is the expression to the power 3/2).

Thus, with this formula, you can, for any function f(x), derive a "curvature function" and study it to investigate how the curvature changes with x, find points of max or min curvature, etc.!

Spring 2003 Exam #2

1. (9 points each) On (a) – (c), evaluate the given integral. If you use the table, be sure to give the number of the integral(s) used.
   a. $\int \frac{4x^2 + 1}{4x^3 + 3x} dx$
   b. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}(e^{\sqrt{x}} - 6)^7} dx$
   c. $\int \frac{3\sqrt{25x^2 + 7}}{8x^2} dx$
2. A car moves along I-70 (an east-west highway) such that its ***velocity*** t hours after noon is given by $v(t) = 3t^2 + 40$ miles per hour.
   a. (5 points) Find the average velocity of the car from 1PM to 3PM.
   b. (5 points) Find the total distance the car traveled between 1PM and 3PM.
   c. (8 points) If the car was 225 miles east of Columbus at 5PM, where will it be at 6:30PM?
3. (9 points) A cylindrical tank 8 feet in diameter is lying on its side and is half full of oil of density 75 pounds per cubic foot. **SET UP** an integral that will find the total force exerted by the oil on one end (e.g., the lid) of the tank. **YOU NEED NOT EVALUATE THE INTEGRAL.**
4. Parts (a) – (d) refer to the region bounded by the graphs $y = x^2$ and $y = -2x + 3$.
   a. (2 points) Sketch the region. Be sure to include points of intersection.
   b. (9 points) **SET UP** an integral that represents the area of the region. ***You need NOT evaluate the integral***.
   c. (9 points) **SET UP** an integral that represents the volume of the solid whose base is the region and whose cross-sections perpendicular to the x-axis are semi-circles. ***You need NOT evaluate the integral***.
   d. (9 points) **SET UP** an integral that represents the volume of the solid generated by rotating the region about the x-axis. ***You need NOT evaluate the integral***.

5. (9 points) The figure below (Note: I do not have the figure, but it was of the kind of the problems in 5.9 and 6.1) represents a tract of land with measurements in feet. A surveyor has measured its width (w) at 50-foot intervals (the values of x shown in the figure) with the following results:

X	0	50	100	150	200	250	300	350	400	450	500	550	600
W	0	165	192	146	63	42	84	155	224	270	267	215	0

Estimate the area of the tract using Simpson's Rule.

5. (8 points) When measuring the lengths, our measuring device is accurate to 6/100 of an inch. If we measure the radius of a ball to be 7 inches, use differentials to estimate how accurate (in cubic inches) our calculation of volume would be if we use the formula $V = \frac{4}{3}\pi r^3$.

Exam #2- Spring 2005

1. (7 points each) In each of the following, evaluate the given integral. If you use a table integral, list its number.

   a. $\int_2^5 (8x^3 - 4x)dx$

   b. $\int \frac{4e^x}{(3e^x - 1)^8} dx$

   c. $\int \frac{6x^2}{\sqrt{7 - 3x^2}} dx$

   d. $\int 8\sin x\sqrt{1 + \cos^2 x}dx$

2. (7 points each) Water flows in and out of a storage tank. A graph of the rate of change $r(t)$ of the volume of water in the tank , in gallons per day, is shown. Assume that the tank is empty when t = 0. In each part, state what the given expression represents in terms of the water and the tank.

   a. $\int_0^4 r(t)dt$

   b. $\int_0^4 |r(t)|dt$

   c. $\frac{1}{4-0}\int_0^4 r(t)dt$

3. (7 points) 3 hours into a trip, a car is located 53 miles east of town. The car's velocity at time t is given by $v(t) = 3t^2 - 12t$ miles per hour, with positive velocity considered to be moving toward the east. Where will the car be 6 hours into the trip?

4. A region in the first quadrant is bounded by the graph of $f(x) = -x^2 + 5x - 4$ and the x-axis.
    a. (6 points) If the density of a flat plate in the shape of the region is given by $\delta(x) = \sqrt{x} + 8$, **SET UP** expressions that will determine the center of mass of the plate. You **NEED NOT** evaluate any integrals.
    b. **(BONUS- 6 points)** Set up an integral that will determine the volume of the solid generated by revolving the region about the line x = -5. You **need not** evaluate the integral.
    c. **(BONUS- 6 points)** Set up an integral that will determine the surface area of the solid generated by revolving the region about the x-axis. You **need not** evaluate the integral.
5. A closed region in the first quadrant is bounded by the graphs of $f(x) = x^2 + x - 1$ and $g(x) = -x^2 + 11x - 9$.
    a. 3 points) Sketch a graph of the region. Be sure to include the x-values of the intersection points.
    b. (6 points) **SET UP** an integral that will determine the area of the region. **YOU NEED NOT EVALUATE THE INTEGRAL.**
    c. (6 points) **SET UP** an integral that will determine the volume of the solid that has the region as its base and cross sections perpendicular to the base and the x-axis that are squares. **YOU NEED NOT EVALUATE THE INTEGRAL**
    d. (6 points) **SET UP** an integral that will determine the volume of the solid generated by revolving the region about the x-axis. **YOU NEED NOT EVALUATE THE INTEGRAL**
    e. (6 points) **SET UP** an expression that will determine the perimeter of the region. **YOU NEED NOT EVALUATE THE INTEGRAL**
6. (6 points) An industrial plant spills pollutant into a lake. The pollutant spreads out to form the pattern shown below. (All distances are in feet). Use Simpson's Rule to estimate the area of the spill.
7. (5 points) An architect wants to build a hemispherical dome with radius 150 meters for an indoor soccer practice field. Use differentials to find the tolerance of error in the measurement of the radius if she wants to be sure that the error in the surface area is at most 2%? FUN FACT: The surface area of a hemisphere is given by: $A = 2\pi r^2$.

Final exam- Spring 2005

1. (6 points each) For parts (a) – (c), find the slope function ($f'(x)$) of the given function.

    a. $f(x) = \tan^{-1}(x^3 - 5)$
    b. $f(x) = e^{\sqrt{x}} \cos(\ln x)$

c. $f(x) = \dfrac{5x}{\sqrt{(7x^2 - 4)}}$

2. (6 points each) For each of the following, evaluate the integral. If you use the table of integrals, indicate the number of the one you are using.

a. $\displaystyle\int_3^6 (x-2)^4\,dx$

b. $\displaystyle\int \frac{5x}{\sqrt{(7x^2-4)}}\,dx$

c. $\displaystyle\int \frac{5x^2}{\sqrt{(7x^2-4)}}\,dx$

d. $\displaystyle\int (\sin x)e^{\cos x}\,dx$

e. $\displaystyle\int e^{3x}\sin(e^{3x}-8)\,dx$

3. (6 points each) Water flows in (positive flow) and out (negative flow) of a tank of water through one pipe at a velocity at time t given by $v(t) = t^2 - 10t + 24$ gallons per hour. Assume that the flow occurs for exactly 9 hours from noon to 9PM (i.e., we only care about t in [0,9]). Note that v(t) becomes zero only at t= 4 and t = 6.

For each of the following, briefly describe (in words and/or symbols) what you would do to $v(t)$ to find the answer. **YOU NEED NOT ACTUALLY FIND THE NUMERICAL ANSWER.**

a. How fast is water flowing at 3PM and in which direction?
b. Is the rate of flow increasing or decreasing at 3PM?
c. At what time is water flowing fastest out of the tank and at what time is water flowing fastest into the tank over the 9 hours from noon to 9PM (noon and 9PM inclusive)?
d. At what time is there the most water in the tank?
e. What is the average velocity of the water from 3PM to 6PM?
f. By how much did the amount of water in the tank change from 2PM to 7PM?
g. How much total water flowed through pipe from 1PM to 7PM?
h. Suppose that at 8PM, there are 100 gallons of water in the tank. How much water was in the tank at 6:15PM?

4. (6 points each) Suppose for a function $f(x)$, the graph of its derivative, $f'(x)$, is increasing on $(-\infty,5)$ and decreasing on $(5,\infty)$. Also, $f'(-3)$ and $f'(8)$ are zero (and no other value of x makes $f'(x)$ zero). Assume that $f(x)$ and all its derivatives have no breaks or vertical asymptotes in their graphs. You may want to sketch the graph of $f'(x)$ before proceeding.

a. Find the intervals where $f(x)$ is increasing and the intervals for which it is decreasing. Also, find the x-coordinates of any relative maxima and/or minima for $f(x)$.
b. Find the intervals where $f(x)$ is concave up and the intervals for which it is concave down. Also, find the x-coordinates of any points of inflection for $f(x)$.
c. Assuming that the only x-intercepts for $f(x)$ are –6, -1, and 11 and the y-intercept for $f(x)$ is 1, sketch a graph of $f(x)$ using the information you found in parts (a) and (b).

5. (9 points) A farmer has 600 yards of fencing with which to build a rectangular corral. Some of the fencing will be used to construct two internal divider fences, both parallel to the same two sides of the corral. Use calculus to find the maximum possible total area of such a corral. Also, use the second derivative test to verify the area you found is a maximum.

Final Exam: Winter 2004

1. (7 points each) For each of the following functions, find its derivative. You need not simplify.

a. $f(x) = (\cos x)\sqrt{\ln x}$

b. $f(x) = e^{\sin^{-1} x} + x^{\pi} + e^{\sqrt{3}}$

c. $f(x) = \dfrac{x^5}{(x^6 - 4)^{20}}$

2. (7 points each) For each of the following, evaluate the integral. If you use the table of integrals, indicate the number of the one you are using.

a. $\int \frac{x^5}{(x^6-4)^{20}}dx$

b. $\int \frac{\sqrt{64-9x^2}}{13x^2}dx$

c. $\int (\sin x)e^{\cos x}\,dx$

3. Given the following table:

x	$f(x)=\frac{1}{1+x^2}$
0	1
1/6	.97297
1/3	.9
1/2	.8
2/3	.69231
5/6	.59016
1	.5

a. (6 points) Use Simpson's rule with 6 subintervals to estimate the area under the curve $f(x)=\frac{1}{1+x^2}$ and above the x-axis on [0,1].

b. (**BONUS: 4 points)** Multiply your result by 4 and describe why the (correct) result occurs.

4. (6 points each) Water flows in (positive flow) and out (negative flow) of a tank of water through one pipe at a velocity at time t given by $v(t)=t^2-10t+24$ gallons per hour. Assume that the flow occurs for exactly 9 hours from noon to 9PM (i.e., we only care about t in [0,9]). Note that v(t) becomes zero only at t= 4 and t = 6.

For each of the following, briefly describe (in words and/or symbols) what you would do to $v(t)$ to find the answer. **YOU NEED NOT ACTUALLY FIND THE NUMERICAL ANSWER.**

a. How fast is water flowing at 3PM and in which direction?

d. Is the rate of flow increasing or decreasing at 3PM?

e. At what time is water flowing fastest out of the tank and at what time is water flowing fastest into the tank over the 9 hours from noon to 9PM (noon and 9PM inclusive)?

f. What is the average velocity of the water from 3PM to 6PM?
g. How much did the amount of water in the tank change from 2PM to 7PM?
h. How much total water flowed through pipe from 1PM to 7PM?
g. Suppose that at 8PM, there are 100 gallons of water in the tank. How much water was in the tank at 6:15PM?

5. Suppose for a function $f(x)$, the graph of its derivative, $f'(x)$, is increasing on $(-\infty,5)$ and decreasing on $(5,\infty)$. Also, $f'(-3)$ and $f'(8)$ are zero (and no other value of x makes $f'(x)$ zero). Assume that $f(x)$ and all its derivatives have no breaks or vertical asymptotes in their graphs.

a. (7 points) Find the intervals where $f(x)$ is increasing and the intervals for which it is decreasing. Also, find the x-coordinates of any relative maxima and/or minima for $f(x)$.
b. (7 points) Find the intervals where $f(x)$ is concave up and the intervals for which it is concave down. Also, find the x-coordinates of any points of inflection for $f(x)$.
c. (9 points) Assuming that the only x-intercepts for $f(x)$ are –6, -1, and 11 and the y-intercept for $f(x)$ is 1, sketch a graph of $f(x)$ using the information you found in parts (a) and (b).

6. (10 points) The U. S. Postal Service will accept a box for domestic shipment only if the sum of its length plus the girth (girth means "distance around") does not exceed 108 inches. What dimensions will give a box with a square end the largest possible volume and allow it to meet postal regulations? (Notes: The volume of a box is the product of its length, width, and height. Also, assume that the girth measures the distance around the square end of the box). Use the second derivative test to verify that you do find a maximum.

7. (9 points) Suppose that a shark tank at a zoo is constructed with a window that resembles the region bounded above by the x-axis and bounded below by the graph $y = x^2 - 25$. Assume the top of the window is 3 feet below the water surface. **SET UP** an integral that represents the force exerted by the seawater on the window. Assume the weight density constant for seawater is 64 pounds per cubic foot. Also, neglect any motion by the sharks! ***You need not evaluate the integral.***

8. Each of the following refers to the region bounded above by $y = -x^2 + 4$ and below by $y = 3x$.

a. (2 points) Sketch the region. Be sure to include points of intersection.

b. (6 points) **SET UP** an integral that represents the volume of the solid with cross sections that are squares whose bases are on the region perpendicular to the x-axis. ***You need not evaluate the integral.***
b. (10 points) Suppose that the region is made of material such that the density of material at x is given by $\delta(x) = x^4$. **SET UP** to find the center of gravity of the region (i.e., write the quotients necessary to

compute each coordinate of the center, but ***you need not evaluate the integrals.***)

Final Exam: Spring 2003

1. (8 points each) For parts (a) – (c), find the slope function ($f'(x)$) of the given function.

a. $f(x) = \tan^{-1}(\sqrt{x^3 - 5})$

b. $f(x) = e^{\sqrt{x}} \cos(\ln x)$

c. $f(x) = (\frac{11}{3x} - 4x)^e + e^{(\frac{11}{3x} - 4x)}$

2. (8 points each) For parts (a) – (d), find $f(x)$ given its slope function. If you use a table integral, be sure to state its number.

a. $f'(x) = \tan x(\ln(\cos x))^4$ (Hint: Write $\tan x = \frac{\sin x}{\cos x}$)

b. $f'(x) = \frac{11e^{7-5x}}{9 - e^{7-5x}}$

c. $f'(x) = \frac{2}{(25 + 3x^2)^2}$

d. $f'(x) = e^{7x} \cos(4x)$

3. (5 points) Let $f(x) = x^2 - 5x - 8$. Find two points that lie on the line that is tangent to $f(x)$ at x = 10.

4. A vehicle travels on an east-west road with its ***velocity*** at time t hours after noon given by $v(t) = -t^2 + 10t - 21$. The vehicle travels continuously until midnight.

   a. (6 points) During which times did the vehicle travel forward (east)? During which times did it travel backwards (west)?
   b. (7 points) For the times it was traveling forward, find the maximum forward velocity
   c. (7 points) **Set up** an expression(s) that would find the total distance the car traveled forward. ***You need not evaluate the expression.***
   d. (7 points) **Set up** an expression(s) that would find the total distance the car traveled backward. ***You need not evaluate the expression.***

e. (7 points) **Set up** an expression(s) that will answer where the car would be (distance and direction) at midnight relative to its original (noon) starting position. ***You need not evaluate the expression.***

5. Suppose $f(x)$ is a polynomial function that is decreasing on $(-\infty,-1)\cup(4,\infty)$ and increasing on $(-1,4)$. Also, the only x-intercepts of f are at x= –3, 2, and 7.

   a. (9 points) Sketch a graph of $f'(x)$.

   b. (14 points) Sketch a graph of $\int f(x)dx$ if this graph has its only x-intercepts at x = -5 and x = 10 along with y-intercept 2. State the x-coordinates at which this graph has relative extrema (state each as a relative max or min) and inflection points. (Hint: you have enough information make sign charts for both the function's first derivative and second derivative).

6. (10 points) A farmer has 600 yards of fencing with which to build a rectangular corral. Some of the fencing will be used to construct two internal divider fences, both parallel to the same two sides of the corral. Use calculus to find the maximum possible total area of such a corral. Also, use the second derivative test to verify the area you found is a maximum.

7. (10 points) A rope is to be connected between two poles in such a way that if the x-axis represents the ground, the rope will resemble the curve $y = x^4 - 3x + 5$ if the poles are at x = -1 and x = 3. **SET UP** the integral that represents the amount of rope needed (neglect the amount needed to tie or connect the rope at the ends). **YOU NEED NOT EVALUATE THE INTEGRAL**

8. (12 points) Find the point on the surface of a solid wood (constant density) table top at which you would connect a leg to perfectly balance the figure. The figure resembles the region bounded by the curve $y = 60x^2$, the x-axis, and the lines x = 0 and x = 2.

Autumn 2005- Final Exam

1. (5 points each) Evaluate the following integrals. If you use a table integral, indicate which number you are using.

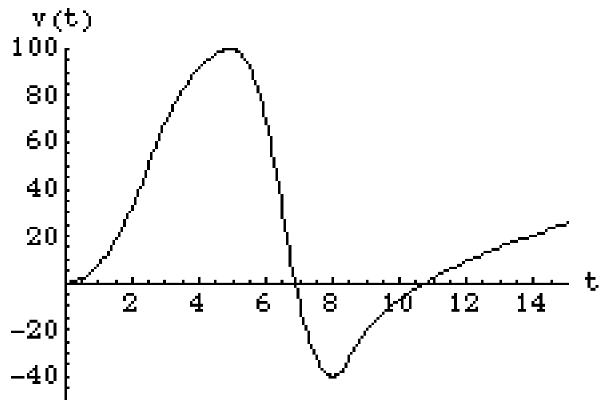
   a. $\int_2^6 (2x-1)^3 dx$

   b. $\int \frac{(\sqrt{x}-7)^9}{5\sqrt{x}} dx$

   c. $\int \frac{7x}{\sqrt{4x^2-9}} dx$

d. $\int \frac{7x^2}{\sqrt{4x^2-9}}\,dx$

2. (5 points each) A driver goes out to test his car on a deserted stretch of road. The graph of his velocity v(t) for the first 15 minutes of his journey is given below, with t in minutes.



a. What does $\int_2^8 v(t)dt$ mean in terms of the car?

b. What does $\int_2^8 |v(t)|dt$ mean in terms of the car?

c. What significant event occurs with the car at t = 7?

d. What significant event occurs with the car at t=5?

e. At what time(s) could the car be the furthest away from its starting position? Briefly explain.

f. At what time(s) could the acceleration of the car be the greatest? Briefly explain.

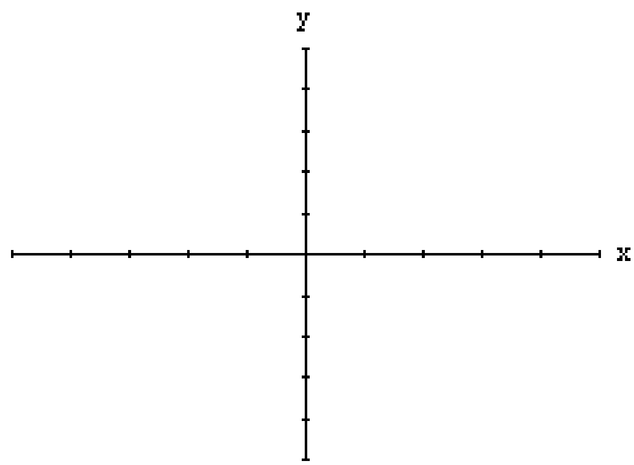
3. (7 points) Assume a brand new car (odometer starts at zero) travels in the same direction. If the acceleration function of the car is a(t) = 6t, with t in hours and distance in miles, and 2 hours into the drive the car is traveling 35 miles per hour, how far away is the car from its starting point after 6 hours?

4. (7 points) Oil is leaking out of a tanker damaged at sea. The damage to the tanker is worsening as evidenced by the increased rate of leakage, recorded every two hours in the following table:

Time (Hours)	0	2	4	6	8	10	12	14	16
Leakage (gal/hr)	50	70	97	136	190	265	369	516	720

Use Simpson's Rule to estimate the total quantity of oil that leaked out of the tanker over the 16 hours.

5. (5 points each) Each of the following refers to the region in the first quadrant bounded above by $y = -2x + 15$ and below by $y = x^2$. You may want to sketch the region here, including intersections.



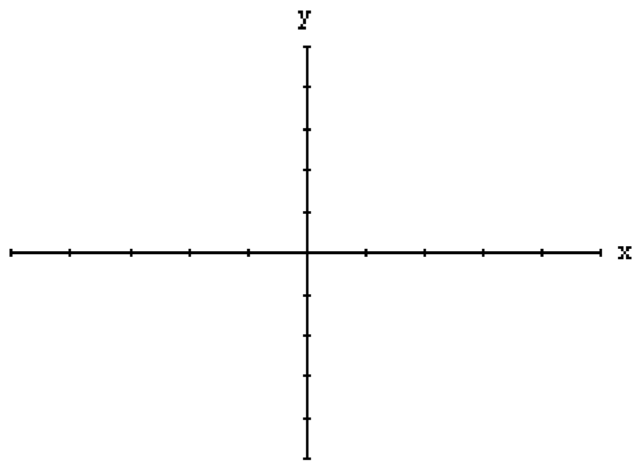
a. **SET UP** an integral that represents the area of the region. ***You need not evaluate the integral.***

b. **SET UP** an integral that represents the volume of the solid with cross sections that are squares whose bases are on the region perpendicular to the x – axis. ***You need not evaluate the integral***.

c. **SET UP** an integral that represents the volume of the solid generated by rotating the region about the x-axis. ***You need not evaluate the integral***.

6. (5 points) Find the center of mass of the region in the first quadrant bounded below by the x-axis and above by the curve $y = x^3$ on (1, 4). Assume the region is of uniform density.

7. (8 points) Sketch the graph of the (continuous) function f(x) whose derivative is positive on $(-\infty, 3)$ and $(6, \infty)$, and negative on $(3, 6)$. Also, the second derivative is negative on $(-\infty, 5)$ and positive on $(5, \infty)$. Note: f(-1) = 0, f(0) = 3, f(3)=4, f(5)=2, and f(6)=1. Be sure to scale your axes and note any relative extrema (max and min) and inflection points.



8. (8 points) A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 800 meters of wire at your disposal, use calculus to find the largest area you can enclose and its dimensions. Also, use calculus to verify you have found the largest possible area.

Final Exam- Winter 2006

1. (6 points each) For each of the following, evaluate the given integral. If you use a table integral, be sure to state its number.

   a. $\int_2^4 (3x-5)^5 dx$

   b. $\int \frac{(e^x - 5)}{(e^x - 5x)^3} dx$

   c. $\int 5x\sqrt{7x^2 - 9}\, dx$

   d. $\int \frac{\sqrt{5x^2 - 7}}{4x} dx$

2. (10 points) A graphing artist is designing a rectangular poster, which is to have margins of 2 inches at the top and along each side, and a 3 inch margin at the bottom. In order to save expenses, she wants the total area of the poster to be as small as possible, but the printed area (the part inside the margins) has to be 180 square inches. What dimensions of the poster will minimize the total area? Be sure to include evidence (i.e., first or second derivative test) that your dimensions do give a minimal area.

3. The following problems refer to the region bounded by the curves $f(x) = x^2 - 7x + 15$ and $g(x) = -x^2 + 10$.

a. (0 points, but should be done) Sketch the region. Make sure to include points of intersection.

b. (5 points) **SET UP** an expression involving integral(s) that represents the area of the region. ***You need NOT evaluate the integral(s).***

c. (5 points) **SET UP** an expression involving an integral(s) that represents the volume of a solid that has the region as its base and cross-sections perpendicular to the x-axis that are squares. ***You need NOT evaluate the integral(s)***.

d. (5 points) **SET UP** an expression involving an integral(s) that represents the volume of the solid obtained by rotating the region about the x-axis. ***You need NOT evaluate the integral(s)***.

e. (5 points) **SET UP** an expression involving an integral(s) that represents the perimeter of the region. ***You need NOT evaluate the integral(s).***

4. (5points) **SET UP** expressions involving integrals that will find the x and y coordinates of the center of mass of the region in the first quadrant bounded by the graph of $f(x) = -x^2 + 4$. You may assume constant density. ***You need NOT evaluate the integrals***.

5. (5 points) A power plant generates electricity by burning oil, from which come pollutants. Measurements are taken at the end of each month determining the rate at which pollutants are released into the atmosphere, which are recorded as follows:

Month	Jan.	Feb.	March	April	May	June	July	August	Sept.	Oct.	Nov.	Dec.	Jan.
Rate of pollutant Release (tons per month)	6	7.5	8.1	18.2	13.5	15.6	18.9	21	24.3	25.5	26.7	31.2	40

Assuming that the months are the same number of days, use Simpson's Rule to estimate the total tonnage of pollutants released over the course of the year. (Note that we are counting by months: January = Month #1, February = Month #2, etc.)

6. (5 points each) Howard the crazy runner loves to run up and down the straight track at the local high school for 20 minutes starting at noon. Howard gets a running start and doesn't stop until a little after the 20 minutes are up. Assuming a positive direction is east and a negative direction is west, Howard's velocity at time t minutes after noon (in feet per minute) is given by $v(t)$. Assuming that you DO NOT have the graph of $v(t)$ (you only have the formula to work with), describe what you would do to answer the following **(You need not carry out the steps you describe)**:

   a. How fast is Howard going at 5 minutes after noon?

   b. At what time from noon until 20 minutes after noon is he going the fastest?

   c. How far did Howard run during the 20 minutes (noon until 20 after noon)?

   d. Where is Howard at 20 minutes after noon relative to where he started?

   e. Is Howard speeding up or slowing down at 7 minutes after noon?

   f. If, at noon, Howard is 35 feet east of the start of the track, where is he at 8 minutes after noon?

7. (6 points) A pendulum of length L has a period $P(L) = \frac{1}{2}\sqrt{L}$. (A pendulum's period is how long the pendulum takes to go from left to right (i.e., "tick-tock").). A certain pendulum is designed to be 9 inches long so that the period is 1.5 seconds. Of course, measurement errors occur in the accuracy of the length of the actual pendulum. Use differentials to estimate how close L needs to be to the 9 inches so that the period is within .001 seconds of 1.5 seconds