Math 117

Study Guide for Exam #1

Structure of the Exam

0 to 1 problem of finding the derivative of a function by definition (most likely a polynomial)
3 to 4 problems of finding the derivative of functions by rule (product, quotient, chain, etc.). You'll need not simplify.
1 to 3 problems of interpreting an English statement in terms of calculus or vice versa, perhaps involving finding key points or intervals (e.g., "when is it moving forward or backward").
1 to 2 problems of finding tangent line to a curve and/or points on that line.
1 to 3 problems of how calculus gives us the features of the graph of or other details of a function (and/or drawing the graph of the derivative given the graph of the function or vice versa)

Study Problems

From the Textbook:
p. 95: 5
p. 145: 3, 15, 16, 21, 28
p. 154: 28, 29
p. 165: 4-11, 37-39
p. 173: 3, 5, 6
p. 174: 25-28
p. 176: 25, 27, 32-34, 38, 40-42, 45-46
p. 191: 11, 18, 41, 45, 46
p. 210: 1-4, 6-8
p. 219: 33
p. 228: 7-32
p. 239: 29-35
p. 252: 3, 27, 29, 30
p. 255: 1-12, 14-15, 17-20, 22-23, 25-34, 35-37, 39-40, 43, 64, 68, 72

Also:

1. Suppose 5 mg of a drug is injected into the bloodstream. Let f(t) be the amount present in the bloodstream after t hours. Interpret what $f(3) = 2$ and $f'(3) = -.5$ means in this situation (be sure to include units).
2. The temperature, T in degrees Fahrenheit, of a cold yam placed in a hot oven is given by $T = f(t)$, where t is the time in minutes since the yam was put in the oven.
    a. What is the sign of $f'(t)$? Why?
    b. What are the units of $f'(20)$. What is the practical meaning of the statement $f'(20) = 2$?
3. The yield, Y, of an apple orchard (measured in bushels of apples per acre) is a function of the amount of fertilizer, x, in pounds used per acre. Suppose that $Y = f(x) = 320 + 140x - 10x^2$.
    a. What is the yield if 5 pounds of fertilizer is used per acre?

b. Find $f'(5)$. Give units with your answer and interpret it in terms of apples and fertilizer.
c. Given your answer to part (b), should more or less fertilizer be used if we are currently using 5 pounds per acre? Explain.

4. In 1774, Captain James Cook left 10 rabbits on a small Pacific island. The rabbit population is approximated by $P(t) = \frac{2000}{1+e^{5.3-0.4t}}$, with t measured in years since 1774. Using a graphing calculator (or we would give you the graph),
   a. Graph P. Does the population level off?
   b. Estimate when the rabbit population grew most rapidly. How large was the population at that time?
   c. Find the inflection point on the graph and explain its significance for the rabbit population.
   d. What natural causes could lead to the shape of the graph of P?
5. When I got up in the morning, I put on only a light jacket because, although the temperature was dropping, it seemed that the temperature would not go much lower. But I was wrong. Around noon a northerly wind blew up and the temperature began to drop faster and faster. The worst was around 6PM when, fortunately, the temperature started going back up.
   a. When was there a time when the derivative of the temperature function (function of time) equal to zero?
   b. When was there an inflection point in the graph of temperature as a function of time?
6. Assuming that f, f' and f'' are defined for all values of x, at what points on the graph of f' would you find values of x for which f might have inflection points?
7. For the function in the graph on a separate page, at what labeled points is the slope of the graph positive? Negative? At what labeled point does the graph have the greatest (most positive) slope? The least (most negative) slope? Answer the same by replacing "slope" with "derivative".
8. For the function f, given on the graph on a separate page,
   a. Sketch $f'(x)$.
   b. Where does $f'(x)$ change its sign?
   c. Where does $f'(x)$ have local maxima or minima?
9. In an 8-second test run, a vehicle accelerates for several seconds and then decelerates. The function s(t) gives the number of feet traveled after t seconds and is graphed, along with its first and second derivatives on a separate page. Be sure to include units in answering the following questions:
   a. How far has the vehicle traveled after 3.5 seconds/
   b. What is the velocity at 2 seconds?
   c. What is the acceleration at 1 second?
   d. When will the vehicle have traveled 120 feet?
   e. When will the vehicle be traveling at a rate of 20 feet per second?
   f. What is the greatest velocity and at what time is the greatest velocity reached? How far has the vehicle traveled at this time?
10. A vehicle moving along a straight road has distance $f(t)$ from its starting point at time t. Which of the graphs on a separate page could be $f'(t)$ for the following scenarios? Assume the scales on the vertical axes are all the same.
    a. A bus on a popular route, with no traffic.
    b. A car with no traffic and all green lights.
    c. A car in heavy traffic conditions.

11. Given $x = f(t) = t^3 - 9t^2 + 15t + 10$ describes the motion of a particle with x representing feet and t representing seconds (t bigger than or equal to 0),
    a. What is its velocity after 2 seconds?
    b. When is the particle at rest?
    c. During what times is the particle moving forward? Backward?
    d. Is the particle moving forward or backward when it is moving the fastest? What is the velocity then?
12. A ladder 10 feet long rests against a vertical wall. Let θ be the angle between the top of the ladder and the wall and let x be the distance from the bottom of the ladder to the wall. If the bottom of the ladder slides away from the wall, how fast does x change with respect to θ when $\theta = \pi/3$?
13. Assuming f and f' are defined for all values of x, what would you look for on the graph of f' to see on what intervals f is increasing or decreasing? At what values of x would you look at on the graph of f'' to see where f had a maximum or minimum? How would you tell from the graph of f' if those values of x gave a max, min, or neither for f?

Old Exam Problems (Winter 2003):

1. If $f(x) = x^2 + 5x - 8$, find $f'(x)$ by definition.

2. For each of parts (a)-(d), find the slope function for $f(x)$. You need not simplify your answer.
    a. $f(x) = x^3 e^{x^4}$
    b. $f(x) = \sin^2(3x^5 - 7x)$
    c. $f(x) = \tan^{-1}(x^2 - 4x)$
    d. $f(x) = \dfrac{\ln(3x^6 + 8)}{\sqrt{x^4 + 9}}$

3. A disease is spreading in such a way that after t weeks, it is affecting

$$N(t) = -t^3 + 6t^2 \text{ hundred people. } (t > 0)$$

    a. Find $N'(3)$. Is this good news or bad news? Explain.
    b. Find $N''(3)$. Is this good news or bad news? Explain.
    c. At what time will the most people be affected? How many people will be affected at that time?
    d. When will the disease be wiped out?
4. The graph of a function f is given below. Sketch a graph of f'. Make sure to clearly mark any x-intercepts. (Note: There are similar problems in the text. I no longer have a picture of the graph I gave the students on this exam).

5. **(Not on Exam #1 for Stewart Text)** Sketch the graph of $f(x) = x^4 - 2x^3$. Be sure to include (and label) any x-intercepts, relative extrema, and points of inflection (found using calculus techniques). When sketching, be careful to include the shape of the graph with respect to increasing/decreasing and its concavity. YOU MAY NOT USE YOUR GRAPHING CALCULATOR EXCEPT AS A CHECKING DEVICE.
6. **(Not on Exam #1 for Stewart Text)** Given $f(x) = 2x^3 - 3x^2 - 12x + 5$, find the absolute maximum and absolute minimum values of f on [-2,4].

7. **(Not on Exam #1 for Stewart Text)** A rectangular box-shaped house is to have a square floor. Three times as much heat per square foot enters through the roof as through the walls. What dimensions should the house be if it is to enclose a volume of 12000 cubic feet and minimize heat entry? (Assume no heat enters through the floor and assume that heat is proportional to area with proportionality constant 1).

Exam #1- Winter 2004

1. (6 points) If $f(x) = x^2 - 5x + 13$, find its slope function by definition.

2. (5 points each) For each of the following functions, find its derivative. You need not simplify.

   a. $f(x) = x^4 \ln x + 3e^x$

   b. $f(x) = (\sin x)^e + e^{(\sin x)}$

   c. $f(x) = \sin^{-1}(x^3 - 5x + 6)$

   d. $f(x) = \dfrac{\sqrt{3x^5 - 5x + 2}}{x - 5}$

3. A frozen dinner is placed in an already-heated oven for the recommended 20 minutes. The graph of the behavior of the dinner's temperature as a function of time (call it $f(t)$.) is given on the back page.

   a. (5 points) Which is greater, $f(3)$ or $f(17)$? Briefly explain in terms of the temperature.

   b. (5 points) Which is greater, $f'(3)$ or $f'(17)$? Briefly explain in terms of the temperature.

4.. A section of roller coaster is in the shape of $y = \frac{1}{5}x^5 - \frac{5}{3}x^3 - 36x + 250$ for x in [-4, 4]. Assume riders go left to right (as x increases). Assume you do not have access to a picture of the coaster.

   (**HELPFUL NOTES**: The derivative turns out to be quadratic in form. Also, the x values are *restricted* on a closed interval!)

   a. (4 points) What information could you find to be able to tell if the coaster going up or down at x=2?

   b. (4 points) What information could you find to be able to tell if the coaster is speeding up or slowing down at x=2?

   c. (6 points) Find the highest and lowest points on the coaster for x in [-4,4].

   d. (6 points) Find where the coaster is steepest (regardless of direction) for x in [-4, 4].

5. Given the following graph (on a separate page) of $y = f(x)$ and that $f''(x)$ is zero only at x=3,

   1. (6 points) Sketch the graph of $f'(x)$. Be sure to include any x-intercepts of that graph.
   2. (6 points) Sketch the graph of $f''(x)$. Be sure to include any x-intercepts of that graph.

6.. **(Not on Exam #1 for Stewart Text)** Given $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$, follow the steps to ward sketching a graph of the function that includes its x-intercepts, relative extrema, and inflection points.

(**FUN FACTS:** $f(x) = 0$ only when x=-1, x=.612, and x = 2.72. Also, $\frac{2 \pm \sqrt{28}}{6} = 1.21$ and -.549.)

a. (2 points) Find any x-intercepts of f. You need not find where f is above or below the x-axis.

b. (6 points) Find any critical values of f (along with relative max and mins) and for what x-values f is increasing or decreasing.

c. (6 points) Find any points of inflection of f and for what x-values f is concave up or concave down.

d. (6 points) Using only the information you found in parts (a), (b), and (c), sketch the graph of f. Be sure to include any x-intercepts, relative extrema, and points of inflection. (NOTE: Your graph should reflect your work and results above. Your graphing calculator can only be used as a checking device).

7**.** **(Not on Exam #1 for Stewart Text)** (10 points) A graphing artist is designing a rectangular poster, which is to have margins of 2 inches at the top and along each side, and a 3 inch margin at the bottom. In order to save expenses, she wants the total area of the poster to be as small as possible, but the printed area (the part inside the margins) has to be 180 square inches. What dimensions of the poster will minimize the total area? Be sure to include evidence that your dimensions do give a minimal area.

Exam #1- Spring 2005

1. Suppose $f(x)$ is a function that describes what the average person should expect for a percentage grade on an exam if he or she studies for x hours. Briefly describe the following equations in terms of hours studied and grades (be sure to include units to go with the numbers!)
   a. (4 points) $f(5) = 73$
   b. (8 points) $f'(28) = -4$
   c. (10 points) $f'(10) > 0$ and $f''(10) = 0$ (with $f''(x) > 0$ for x< 10 and $f''(x) < 0$ for x>10).

2. Given the following graph (on a separate page) of $y = f(x)$, find the following information.
   a. (4 points) Find x-values where $f'(x) = 0$.
   b. (7 points) Find intervals for which $f'(x)$ is positive and negative.
   c. (4 points) Find x-values where $f''(x) = 0$
   d. (7 points) Find intervals for where $f''(x)$ is positive and negative.

3. (10 points each) In each of the following, find the derivative of the given function. You need not simplify.
   a. $f(x) = \frac{e^x}{\sqrt{x^3 + 5}}$
   b. $f(x) = \ln(5x)\tan^{-1}(4x)$
   c. $f(x) = \sin^3(x^4 + 7x)$

(There were other problems that are covered on your final exam, but not on this exam for the Stewart text)

Midterm Exam – Autumn 2005

1. (7 points) If $f(x) = 4x^2 - 3x + 173$, find $f'(x)$ by the limit definition.

2. (6 points each) In each of the following, find $f'(x)$ by any means. You need not simplify your answer.

   a. $f(x) = \dfrac{\cos(5x)}{\sqrt{x^4 - 6x + 4}}$

   b. $f(x) = x^3 \sin^{-1}(7x)$

   c. $f(x) = e^{(3x^2+5)} - (3x^2 + 5)^8$

3. (5 points each) Suppose you just bought a bag of cookies and put it on the table. If $C(t)$ represents the number of cookies in the bag at time t (in minutes), give an equation that represents the given statement using either $C, C',$ or C'' (That is, your answers should look something like $C'(7) = 8$).

   a. The bag starts off with 160 cookies.

   b. 5 minutes after putting the bag on the table, cookies are being eaten at a rate of 4 cookies per minute!

   c. 12 minutes after putting the bag on the table, the cookies are being eaten at a steady rate.

4. (5 points each) Suppose the height of a person is given by $H(t)$, where t is in years.

   a. Do you expect $H(3)$ to be positive or negative? Explain.

   b. Do you expect $H'(3)$ to be positive or negative? Explain.

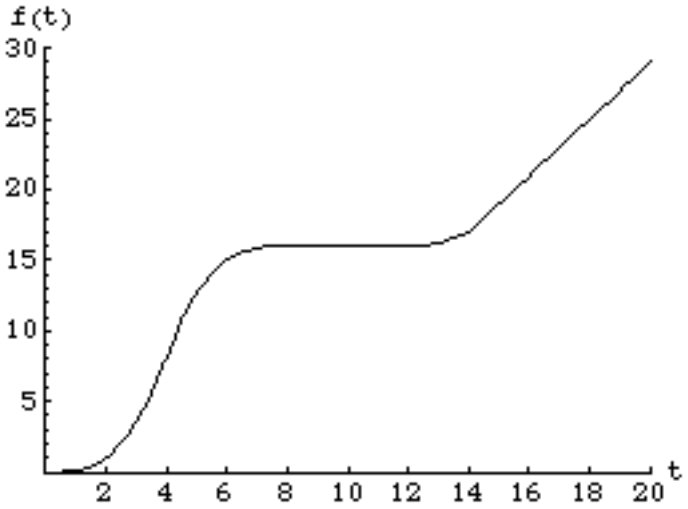
   c. Do you expect $H''(20)$ to be positive or negative? Explain.

5. (5 points each) Given that the distance, in miles, of an object from "home" is given by $f(t) = t^3 - 12t^2 + 36t + 5$, where t is in hours after noon, answer the following. Assume "positive" distance is east of home and "negative" distance is west of home.

   a. Find the velocity at 3PM.
   b. Find the time(s) when the object turns around (changes direction).
   c. Is the object speeding up or slowing down at 1PM?

6. (5 points each) The following graph describes the trip a car takes from home. $f(t)$ is the distance that the car is away from home (in miles) after t minutes of travel. Approximate the time(s) at which:

   a. The car is furthest from home.
   b. The car is traveling the fastest.
   c. The car is stopped.
   d. The car is traveling on cruise control (i.e., a constant speed above zero)



7. (10 points) Given the following graph of $f(x)$, sketch a graph of $f'(x)$.