Math 117
Spring 2006

Name: _____

Final Exam - Form C

Instructions: You have 1 hour and 48 minutes to complete this exam. It is worth a total of 100 points and the value of each question is listed with each question. You may use two 8.5"x11" sheets of notes (both sides). You may use a calculator which has no symbolic algebra or calculus capabilities (eg. not a TI-89, TI-92). Please write clearly and make sure to justify your answers.

(1.)(15 pts.) Evaluate the following integrals. If you use a table integral, please indicate which one that you are using.

(a.) $\int_0^1 x^6(1-x^7)^{99}\,dx$

(b.) $\int \frac{27x^2-60x}{3x^3-10x^2}\,dx$

(c.) $\int \frac{7x^2}{\sqrt{6-4x^2}}\,dx$

(2.)(30 pts.) A car starts at noon and travels with the velocity $v(t)$ shown in the graph below. A truck starts at 1 PM from the same place that the car did and travels at a constant velocity of 50 mph. In each of the following, explain how you found your answer.

(a.) At what time is the car traveling the fastest and what is its velocity at that time?

(b.) How fast is the distance between the car and the truck changing at 4PM?

(c.) What does $\int_0^{10} v(t)\, dt$ mean with respect to both the graph and the car?

(d.) During the time that the car is ahead of the truck, when is the distance between them the greatest?

(e.). When does the truck overtake the car?

(f.) When does the car have its greatest acceleration?

(3.)(15 pts.) Consider the region bounded by the curves $y = x^2 + 2x + 5$ and $y = -x^2 + 10x + 15$.

(a.) Sketch the region. Make sure to include points of intersection.

(b.) SET UP an expression involving integral(s) that represents the area of the region. You need NOT evaluate the integral(s).

(c.) SET UP an expression involving an integral(s) that represents the volume of a solid that has the region as its base and cross-sections perpendicular to the x-axis that are isosceles right triangles. You need NOT evaluate the integral(s).

(d.) SET UP an expression involving an integral(s) that represents the volume of the solid obtained by rotating the region about the x-axis. You need NOT evaluate the integral(s).

(4.)(12 pts.) A rough approximation of the growth of a bacteria in a culture dish is given by $m'(t) = \frac{.05}{2\sqrt{t}}$ in grams per hour of biomass after the first hour. At t=1, there are 2.6 grams in the dish.

(a.) What is the mass function $m(t)$?

(b.) By how much did the weight of the bacteria change from t=2 to t=6?

(c.) How many grams are there in the dish at the end of the third hour (i.e., when t=3)?

(d.) When will there be 10 grams in the dish?

(5.)(10 pts.) Suppose that $f(x)$ is a function which satisfies:

$f(x) > 0$ if $x < 1$ or $x > 5$ and $f(x) < 0$ if $1 < x < 5$

$f'(x) > 0$ if $x < -2$ or $3 < x < 8$ and $f'(x) < 0$ if $-2 < x < 3$ or $x > 8$

$f''(x) > 0$ if $0 < x < 6$ and $f''(x) < 0$ if $x < 0$ or $x > 6$

Sketch a graph of the function. Make sure to include the x-coordinates of x-intercepts, maxima and minima, and inflection points. Assume the function and its derivatives do not have any breaks (such as vertical asymptotes) in them.

(6.)(6 pts.) Suppose that $0.2 < x < 4$. Find the values for x such that the difference between x^2 and x^3 are the greatest and where this difference is the smallest.

(7.)(12 pts.) A cardboard box with no top and a square bottom is to have a volume of 729 cubic inches.

(a.) What are the dimensions of the box which uses the least amount of cardboard?

(b.) What is the minimal amount of cardboard?