1. Let \( A = \begin{pmatrix} 3 & 6 & 3 & 6 & 1 \\ 5 & 10 & 4 & 2 & -3 & 6 \\ 7 & 14 & 2 & 0 & -11 & 7 \\ 1 & 2 & 0 & 0 & -1 & 2 \end{pmatrix} \) and find:

(a.) A basis for the row space of \( A \).
(b.) A basis for the column space of \( A \).
(c.) A basis for the nullspace of \( A \). DO NOT USE MATLAB.
(d.) The rank of \( A \).

2. Suppose that \( \mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \), \( \mathbf{v}_2 = \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix} \), \( \mathbf{v}_3 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \) and \( \mathbf{v}_4 = \begin{pmatrix} 2 \\ 2 \\ 8 \end{pmatrix} \). Find a basis for \( \text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4) \).

Note: there is more than one answer.

3. Determine if \( S \) is a subspace of \( V \) where:

(a.) \( V = \mathbb{R}^{2 \times 2} \) and \( S \) is the set of \( 2 \times 2 \) matrices \( A \) with \( \det(A) = 0 \).
(b.) \( V = \mathbb{R}^{2 \times 2} \) and \( S \) is the set of \( 2 \times 2 \) upper triangular matrices.
(c.) \( V = \mathbb{R}^2 \) and \( S = \{(x_1, x_2)^T | |x_1| = |x_2|\} \).
(d.) \( V = \mathbb{P}_2 \) and \( S \) is the set of all polynomials \( p(x) \) in \( V \) such that \( p(1) = 0 \).
(e.) \( V = \mathbb{C}[-1, 1] \) and \( S \) is the set of odd functions in \( V \).

4. Determine if \( 1, e^x \) and \( \cos x \) are linearly independent in \( \mathbb{C}[0, 1] \).

5. Find a basis for the subspace \( S \) of \( V \) where:

(a.) \( V = \mathbb{R}^3 \) and \( S = \{(a - b + c, a + c, a + 2b - c, b - 3c)^T | a, b, c \text{ are real numbers}\} \).
(b.) \( V = \mathbb{C}[0, 1] \) and \( S = \text{Span}(1, \sin 2x, \sin x \cos x) \).
(c.) \( V = \mathbb{P}_4 \) and \( S \) is the set of all polynomials \( p(x) \) in \( V \) with \( p(0) = 0 \) and \( p(1) = 0 \).

6. Let \( A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \) and let \( S = \{B \in \mathbb{R}^{2 \times 2} | AB = BA\} \).

(a.) Show that \( S \) is a subspace of \( \mathbb{R}^{2 \times 2} \).
(b.) Find a basis for \( S \).

7. True or False?

(a.) A linearly independent set can not contain \( \mathbf{0} \).
(b.) A subspace of a vector space must contain \( \mathbf{0} \).
(c.) If \( A \) is an \( m \times n \) matrix, then \( \dim(\text{Col}(A)) + \dim(\text{N}(A)) = m \).
(d.) If \( A \) is an \( m \times n \) matrix, then \( Ax = b \) is consistent if and only if \( b \) is in the column space of \( A \).
(e.) If \( A \) is a singular \( n \times n \) matrix, then the columns of \( A \) form a basis for \( \mathbb{R}^n \).
(f.) A spanning set can never be linearly independent.

8. Prove the following:

(a.) Let \( S \) be the subset of \( \mathbb{R}^{n \times n} \) consisting of matrices \( A \) such that \( A^T = A \). Show that \( S \) is a subspace of \( \mathbb{R}^{n \times n} \). (The matrices in \( S \) are called symmetric matrices.)

(b.) If \( \{\mathbf{v}_1, \ldots, \mathbf{v}_n\} \) are linearly independent vectors in a vector space \( V \), then \( \{\mathbf{v}_2, \ldots, \mathbf{v}_n\} \) do not span \( V \).
(c.) If \( \{\mathbf{v}_1, \ldots, \mathbf{v}_n\} \) are linearly independent vectors in a vector space \( V \) which do not span \( V \), then there is a vector \( \mathbf{v}_0 \) in \( V \) such that \( \{\mathbf{v}_0, \mathbf{v}_1, \ldots, \mathbf{v}_n\} \) is also linearly independent.