Let $A = \begin{pmatrix} 3 & 6 & 3 & 3 & 6 & 1 \\ 5 & 10 & 4 & 2 & -3 & 6 \\ 7 & 14 & 2 & 0 & -11 & 7 \\ 1 & 2 & 0 & 0 & -1 & 2 \end{pmatrix}$ and find:

(a.) A basis for the row space of $A$.
(b.) A basis for the column space of $A$.
(c.) A basis for the nullspace of $A$. DO NOT USE MATLAB.
(d.) The rank of $A$.

(2.) Suppose that $v_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, $v_2 = \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix}$, $v_3 = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$ and $v_4 = \begin{pmatrix} 2 \\ 2 \\ 8 \end{pmatrix}$. Find a basis for $\text{Span}(v_1, v_2, v_3, v_4)$.

Note: there is more than one answer.

(3.) Determine if $S$ is a subspace of $V$ where:
(a.) $V = \mathbb{R}^{2 \times 2}$ and $S$ is the set of $2 \times 2$ matrices $A$ with $\det(A)=0$.
(b.) $V = \mathbb{R}^{2 \times 2}$ and $S$ is the set of $2 \times 2$ upper triangular matrices.
(c.) $V = \mathbb{R}^2$ and $S = \{(x_1, x_2)^T \mid |x_1| = |x_2|\}$.
(d.) $V = \mathbb{P}_3$ and $S$ is the set of all polynomials $p(x)$ in $V$ such that $p(1) = 0$.
(e.) $V = C[-1, 1]$ and $S$ is the set of odd functions in $V$.

(4.) Determine if $1, e^x$ and $\cos x$ are linearly independent in $C[0, 1]$.

(5.) Find a basis for the subspace $S$ of $V$ where:
(a.) $V = \mathbb{R}^4$ and $S = \{(a - b + c, a + c, a + 2b - c, b - 3c)^T \mid a, b, c \text{ are real numbers}\}$.
(b.) $V = \mathbb{P}_2$ and $S = \text{Span}(1, \sin 2x, \sin x \cos x)$.
(c.) $V = \mathbb{P}_4$ and $S$ is the set of all polynomials $p(x)$ in $V$ with $p(0) = 0$ and $p(1) = 0$.

(6.) Let $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ and let $S = \{B \in \mathbb{R}^{2 \times 2} \mid AB = BA\}$.

(a.) Show that $S$ is a subspace of $\mathbb{R}^{2 \times 2}$.
(b.) Find a basis for $S$.

(7.) True or False?
(a.) A linearly independent set can not contain $0$.
(b.) A subspace of a vector space must contain $0$.
(c.) If $A$ is an $m \times n$ matrix, then $\dim(\text{Col}(A)) + \dim(\text{N}(A)) = m$.
(d.) If $A$ is an $m \times n$ matrix, then $Ax = b$ is consistent if and only if $b$ is in the column space of $A$.
(e.) If $A$ is a singular $n \times n$ matrix, then the columns of $A$ form a basis for $\mathbb{R}^n$.
(f.) A spanning set can never be linearly independent.

(8.) Prove the following:
(a.) Let $S$ be the subset of $\mathbb{R}^{n \times n}$ consisting of matrices $A$ such that $A^T = A$. Show that $S$ is a subspace of $\mathbb{R}^{n \times n}$.
(b.) If $\{v_1, \ldots, v_n\}$ are linearly independent vectors in a vector space $V$, then $\{v_2, \ldots, v_n\}$ do not span $V$.
(c.) If $\{v_1, \ldots, v_n\}$ are linearly independent vectors in a vector space $V$ which do not span $V$, then there is a vector $v_0$ in $V$ such that $\{v_0, v_1, \ldots, v_n\}$ is also linearly independent.