Math 571

Winter 2010

Quiz 5

Instructions: This quiz is worth 10 points and the value of of each question is listed with each question. You may use any notes or books but you must work individually. The only computation aid which you may use is MATLAB, unless otherwise indicated. Make sure to write clearly and justify your answers.

(1.)(3 pts.) Let $A = \begin{pmatrix} 3 & 6 & -4 & 1 & -31 & -4 \\ -4 & -8 & -2 & -4 & 10 & -40 \\ -1 & -2 & 2 & 3 & 7 & 24 \\ -5 & -10 & 1 & -1 & 22 & -12 \end{pmatrix}$ and find:

- **(a.)** A basis for the row space of A.
- **(b.)** A basis for the column space of A.
- **(c.)** A basis for the nullspace of A. DO NOT USE MATLAB.
- **(d.)** The rank of A.

(2.)(3 pts.) Let $\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^T * \begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 8 \end{pmatrix} * \mathbf{y}$ where $\mathbf{x}, \mathbf{y}$ are vectors in $\mathbb{R}^3$ and $*$ is the usual matrix multiplication.

- **(a.)** Show that this defines an inner product on $\mathbb{R}^3$.
- **(b.)** If $\mathbf{x} = (1, 2, 1)^T$ and $\mathbf{y} = (-1, 2, 2)^T$, find (with respect to THIS inner product):
  - **(i.)** $\langle \mathbf{x}, \mathbf{y} \rangle$,
  - **(ii.)** $||\mathbf{x}||$ and $||\mathbf{y}||$,
  - **(iii.)** the orthogonal projection of $\mathbf{x}$ onto $\mathbf{y}$ and
  - **(iv.)** the angle between $\mathbf{x}$ and $\mathbf{y}$.

(3.)(2 pts.) Suppose that $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 2 \\ 3 & 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} -3 & 3 & 2 \\ -4 & 1 & 1 \\ 1 & -2 & -2 \end{pmatrix}$. Find

- **(a.)** $\langle A, B \rangle$,
- **(b.)** $||A||$ and $||B||$,
- **(c.)** The angle between A and B and
- **(d.)** The orthogonal projection of A onto the space spanned by B.

(4.)(2 pts.) Let $\langle f(x), g(x) \rangle$ be the usual inner product on $C[0, 1]$. If $f(x) = x^3 + 1$ and $g(x) = 2x$, find:

- **(a.)** $\langle f(x), g(x) \rangle$,
- **(b.)** $||f(x)||$ and $||g(x)||$,
- **(c.)** The angle between $f(x)$ and $g(x)$ and
- **(d.)** The orthogonal projection of $f(x)$ onto the space spanned by $g(x)$.