Review 2

(1.) Let $A = \begin{pmatrix} 3 & 6 & 3 & 3 & 6 & 1 \\ 5 & 10 & 4 & 2 & -3 & 6 \\ 7 & 14 & 2 & 0 & -11 & 7 \\ 1 & 2 & 0 & 0 & -1 & 2 \end{pmatrix}$.

(a.) Find a basis for the row space of A.
(b.) Find a basis for the column space of A.
(c.) Find a basis for the nullspace of A. DO NOT USE MATLAB.
(d.) The rank of A.

(2.) Suppose that $\mathbf{x}_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \mathbf{x}_2 = \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix}, \mathbf{x}_3 = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$ and $\mathbf{x}_4 = \begin{pmatrix} 2 \\ 2 \\ 8 \end{pmatrix}$. Find a basis for $\text{Span}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4)$.
Note: there is more than one answer.

(3.) Determine if $1, e^x$ and $\cos x$ are linearly independent in $C[0, 1]$.

(4.) Find a basis for the subspace S of V where:
(a.) $V = \mathbb{R}^4$ and $S = \{(a - b + c, a + c, a + 2b - c, b - 3c)^T \mid a, b, c \text{ are real numbers } \}$.
(b.) $V = C[0, 1]$ and S=Span$(1, \sin 2x, \sin x \cos x)$.
(c.) $V = \mathbb{P}_4$ and S is the set of all polynomials $p(x)$ in V with $p(0) = 0$ and $p(1) = 0$.

(5.) Let $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ and let $S = \{B \in \mathbb{R}^{2\times2} \mid AB = BA\}$.

(a.) Show that S is a subspace of $\mathbb{R}^{2\times2}$.
(b.) Find a basis for S.

(6.) Let $x_1 = 1, x_2 = 2, x_3 = -1$ and $x_4 = -2$. Then

$$\langle p(x), q(x)\rangle = p(x_1)q(x_1) + p(x_2)q(x_2) + p(x_3)q(x_3) + p(x_4)q(x_4)$$

defines an inner product on $\mathbb{P}_4$. If $p(x) = x^3 - 6x^2 + 2$ and $q(x) = x^3 + x - 1$, find the following with respect to THIS inner product:

(a.) $\langle p(x), q(x)\rangle$, $\quad ||p(x)||$, $\quad ||q(x)||$,
(b.) the orthogonal projection of $p(x)$ onto the line spanned by $q(x)$ and
(c.) the angle between $p(x)$ and $q(x)$.

(7.) True or False?
(a.) A linearly independent set can not contain $\mathbf{0}$.
(b.) If $\mathbf{x}$ and $\mathbf{y}$ are vectors in the inner product space V, then $||\mathbf{x} + \mathbf{y}||^2 = ||\mathbf{x}||^2 + ||\mathbf{y}||^2$.
(c.) If A is an $m \times n$ matrix, then dim(Col(A))+dim(N(A))=m.
(d.) If A is an $m \times n$ matrix, then $A\mathbf{x} = \mathbf{b}$ is consistent if and only if $\mathbf{b}$ is in the column space of A.
(e.) If A is a singular $n \times n$ matrix, then the columns of A form a basis for $\mathbb{R}^n$.
(f.) A spanning set can never be linearly independent.

(8.) Prove the following:
(a.) Let S be the subset of $\mathbb{R}^{n\times n}$ consisting of matrices A such that $A^T = A$. Show that S is a subspace of $\mathbb{R}^{n\times n}$. (The matrices in S are called symmetric matrices.)
(b.) If $\{\mathbf{x}_1, \ldots, \mathbf{x}_n\}$ are linearly independent vectors in a vector space V, then $\{\mathbf{x}_2, \ldots, \mathbf{x}_n\}$ do not span V.
(c.) If $\{\mathbf{x}_1, \ldots, \mathbf{x}_n\}$ are linearly independent vectors in a vector space V which do not span V, then there is a vector $\mathbf{x}_0$ in V such that $\{\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_n\}$ is also linearly independent.
(d.) If $\mathbf{p}$ is the orthogonal projection of $\mathbf{x}$ onto the line spanned by $\mathbf{y}$ then $\mathbf{p}$ and $\mathbf{x} - \mathbf{p}$ are orthogonal.