1. Let $A = \begin{pmatrix} 3 & 6 & 3 & 3 & 6 & 1 \\ 5 & 10 & 4 & 2 & -3 & 6 \\ 7 & 14 & 2 & 0 & -11 & 7 \\ 1 & 2 & 0 & 0 & -1 & 2 \end{pmatrix}$.

(a) Find a basis for the row space of $A$.
(b) Find a basis for the column space of $A$.
(c) Find a basis for the nullspace of $A$. DO NOT USE MATLAB.
(d) The rank of $A$.

2. Suppose that $x_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, $x_2 = \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix}$, $x_3 = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$ and $x_4 = \begin{pmatrix} 2 \\ 2 \\ 8 \end{pmatrix}$. Find a basis for $\text{Span}(x_1, x_2, x_3, x_4)$.

Note: there is more than one answer.

3. Determine if $1, e^x$ and $\cos x$ are linearly independent in $C[0, 1]$.

4. Find a basis for the subspace $S$ of $V$ where:
   (a) $V = \mathbb{R}^4$ and $S = \{(a - b + c, a + c, a + 2b - c, b - 3c)^T \mid a, b, c \text{ are real numbers}\}$.
   (b) $V = C[0, 1]$ and $S = \text{Span}(1, \sin 2x, \sin x \cos x)$.
   (c) $V = \mathbb{P}_4$ and $S$ is the set of all polynomials $p(x)$ in $V$ with $p(0) = 0$ and $p(1) = 0$.

5. Let $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ and let $S = \{B \in \mathbb{R}^{2 \times 2} \mid AB = BA\}$.

(a) Show that $S$ is a subspace of $\mathbb{R}^{2 \times 2}$.
(b) Find a basis for $S$.

6. Let $x_1 = 1, x_2 = 2, x_3 = -1$ and $x_4 = -2$. Then
   $$\langle p(x), q(x) \rangle = p(x_1)q(x_1) + p(x_2)q(x_2) + p(x_3)q(x_3) + p(x_4)q(x_4)$$

defines an inner product on $\mathbb{P}_4$. If $p(x) = x^3 - 6x^2 + 2$ and $q(x) = x^3 + x - 1$, find the following with respect to THIS inner product:
   (a) $\langle p(x), q(x) \rangle$, $||p(x)||$, $||q(x)||$.
   (b) the orthogonal projection of $p(x)$ onto the line spanned by $q(x)$
   (c) the angle between $p(x)$ and $q(x)$.

7. True or False?
   (a) A linearly independent set can not contain $\mathbf{0}$.
   (b) If $\mathbf{x}$ and $\mathbf{y}$ are vectors in the inner product space $V$, then $||\mathbf{x} + \mathbf{y}||^2 = ||\mathbf{x}||^2 + ||\mathbf{y}||^2$.
   (c) If $A$ is an $m \times n$ matrix, then $\text{dim(\text{Col}(A))} + \text{dim(\text{N}(A))} = m$.
   (d) If $A$ is an $m \times n$ matrix, then $A\mathbf{x} = \mathbf{b}$ is consistent if and only if $\mathbf{b}$ is in the column space of $A$.
   (e) If $A$ is a singular $n \times n$ matrix, then the columns of $A$ form a basis for $\mathbb{R}^n$.
   (f) A spanning set can never be linearly independent.

8. Prove the following:
   (a) Let $S$ be the subset of $\mathbb{R}^{n \times n}$ consisting of matrices $A$ such that $A^T = A$. Show that $S$ is a subspace of $\mathbb{R}^{n \times n}$. (The matrices in $S$ are called symmetric matrices.)
   (b) If $\{x_1, \ldots, x_n\}$ are linearly independent vectors in a vector space $V$, then $\{x_2, \ldots, x_n\}$ do not span $V$.
   (c) If $\{x_1, \ldots, x_n\}$ are linearly independent vectors in a vector space $V$ which do not span $V$, then there is a vector $x_0$ in $V$ such that $\{x_0, x_1, \ldots, x_n\}$ is also linearly independent.
   (d) If $\mathbf{p}$ is the orthogonal projection of $\mathbf{x}$ onto the line spanned by $\mathbf{y}$ then $\mathbf{p}$ and $\mathbf{x} - \mathbf{p}$ are orthogonal.