Math 571

Winter 2010

Project 2

Instructions: This project is worth a total of 10 points. You may use any notes or books that you wish but you must work individually. The only computation aid which you may use is MATLAB, unless otherwise indicated. The primary reference for this project are the notes on Markov processes which can be found at: `http://www.math.ohio-state.edu/~husen/teaching/571/markov_1.pdf`. Make sure to write clearly and justify your answers.

(1.)(5 pts.) It is known that a molecule of a particular type can be found in one of three conformations, denoted C_1, C_2 and C_3. These conformations are semi-stable and a given molecule can change from one to another each second. The probability that this happens is given by:

$$C_1 \to C_2; P = 0.76 \qquad C_1 \to C_3; P = 0.07$$

$$C_2 \to C_1; P = 0.58 \qquad C_2 \to C_3; P = 0.17$$

$$C_3 \to C_1; P = 0.20 \qquad C_3 \to C_2; P = 0.52$$

(a.) Find the transition matrix M of this Markov process.

(b.) Suppose that initially there are 250 molecules in the C_1 conformation, 175 in the C_2 conformation and 105 in the C_3 conformation. What is the resulting distribution after 4 seconds?

(c.) Find the steady-state probability vector $\mathbf{x}_s$ of this process.

(2.)(5 pts.) Suppose that one type of atom can exist in a ground state G or be in one of 4 excited states E_1, E_2, E_3 and E_4. Suppose that a collection of these atoms is irradiated by a photon pulse.

If an atom is in the ground state, then it has a probability of 0.15 of being excited to state E_1, a probability of 0.11 of being excited to state E_2 a probability of 0.05 of being excited to state E_3 and a probability of 0.05 of being excited to state E_4.

If it is in state E_1 it has a probability of 0.77 of dropping to the ground state, a probability of 0.05 of being excited to state E_2 a probability of 0.03 of being excited to state E_3 and a probability of 0.03 of being excited to state E_4.

If it is in state E_2 it has a probability of 0.81 of dropping to the ground state, a probability of 0.07 that it transitions to state E_1 and it never transitions to E_3 or E_4.

If it is in state E_3 or E_4 it has a 0.80 probability of dropping to state E_2 and a 0.20 probability of remaining in its current state.

(a.) Find the transition matrix M of this Markov process.

(b.) Find the steady-state probability vector $\mathbf{x}_s$ of this process.

(c.) Approximately how many bursts will it take for this process to be within 4 decimal places of accuracy of the steady-state?