Math 572
Winter 2010

Exam 2 Review

(1.) Find the eigenvalues and corresponding eigenvectors for the following matrices. (Do not use the "`eig()`" function in Matlab):

(a.) $A = \begin{pmatrix} -4 & -6 \\ 3 & 5 \end{pmatrix}$, **(b.)** $A = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 9 & 2 \\ 2 & 4 & 2 \end{pmatrix}$, **(c.)** $A = \begin{pmatrix} 5 & 0 & 0 \\ 3 & 1 & 1 \\ 7 & -2 & 3 \end{pmatrix}$

(2.) Let

$$A = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$$

where a and b are real numbers.

(a.) Find the eigenvalues of A.

(b.) Under what conditions are the eigenvalues of A real?

(3.) Solve the following systems of linear differential equations:

(a.)

$$\begin{aligned} y_1' &= 2y_1 \\ y_2' &= 3y_1 + 5y_2 - 18y_3 \\ y_3' &= 6y_1 + y_2 - y_3 \end{aligned}$$

(b.)

$$\begin{aligned} y_1'' &= -4y_1 + 5y_2 + 5y_1' - y_2' \\ y_2'' &= 5y_2 - 4y_1' + 4y_2' \end{aligned}$$

(4.) Compute $< \mathbf{x}, \mathbf{y} >$, $< \mathbf{y}, \mathbf{x} >$, $||\mathbf{x}||$ and $||\mathbf{y}||$ where:

(a.) $\mathbf{x} = \begin{pmatrix} 2+3i \\ 5-i \end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix} 6-3i \\ 7i \end{pmatrix}$

(b.) $\mathbf{x} = \begin{pmatrix} 2i \\ 5-3i \\ 4+2i \end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix} 3-i \\ 6+i \\ 2+3i \end{pmatrix}$

(5.) Determine if A is diagonalizable where A is given below. If it is diagonalizable, then find a matrix X (which is unitary, if possible) such that $X^{-1}AX$ is diagonal. Also compute e^A in this case.

(a.) $A = \begin{pmatrix} 27 & 28 & -4 \\ -12 & -11 & 2 \\ 21 & 34 & 5 \end{pmatrix}$, **(b.)** $A = \begin{pmatrix} 40 & 5 & 2 \\ -229 & -28 & -12 \\ -188 & -25 & -8 \end{pmatrix}$, **(c.)** $A = \begin{pmatrix} -3 & 8 & 6 & -4 \\ -4 & 2 & -1 & -3 \\ 4 & -1 & -1 & 4 \\ 4 & -3 & -1 & 6 \end{pmatrix}$

(6.) True or False. You do not need to explain you answer.

(a.) A real symmetric matrix is Hermitian.

(b.) Every matrix is similar to an upper triangular matrix.

(c.) If A has distinct eigenvalues, then it is diagonalizable.

(d.) If U is unitary, then $U = U^H$.