Discrete Math: Lecture Notes

Roy Joshua*
Department of Mathematics, Ohio State University, Columbus, Ohio, 43210, USA.
joshua@math.ohio-state.edu

Abstract

What follows is an informal set of lecture notes.

Contents

1 Lecture 15: 4.2  2
Key concepts: Introduction to Induction.

**Basic idea:** Here we want to show that a formula $P(n)$ holds for all integers $n \geq 1$, where $a$ is a fixed positive integer or 0. Induction shows this is true if (i) $P(a)$ is verified to be true and (ii) for all $n \geq a$, $P(n)$ can be shown to imply $P(n+1)$.

**Example 1.1.** Exercise 4 Show that $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ for all $n \geq 1$.

Let $P(n): \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$. Then the LHS of $P(1) = 1$. The RHS of $P(1) = \frac{1 \cdot 2 \cdot 3}{6} = 1$. Therefore, $P(1)$ holds.

Next assume $P(n)$. Then $\sum_{i=1}^{n+1} i^2 = \sum_{i=1}^{n} i^2 + (n+1)^2 = \frac{n(n+1)(2n+1)}{6} + (n+1)^2 = \frac{n(n+1)(2n+1)+6(n+1)^2}{6} = (n+1)(2n^2 + 7n + 6).

On the other hand, $(n+1)(2n+3) = (n+1)(2n^2 + 7n + 6)$. Therefore, we have shown $P(n)$ implies $P(n+1)$.

**Example 1.2.** Exercise 12. Show that $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$ for all $n \geq 1$.

Let $P(n): \sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$. Then teh LHS of $P(1) = \frac{1}{1 \cdot 2} = \frac{1}{2}$. The RHS of $P(1) = \frac{1}{2}$ also. Therefore $P(1)$ holds.

Next we show that $P(n)$ implies $P(n+1)$.

$$\sum_{i=1}^{n+1} \frac{1}{i(i+1)} = \sum_{i=1}^{n} \frac{1}{i(i+1)} + \frac{1}{(n+1)(n+2)} = \frac{n}{n+1} + \frac{1}{(n+1)(n+2)} = \frac{n(n+2)+1}{(n+1)(n+2)} = \frac{(n+1)^2}{(n+1)(n+2)} = \frac{n}{n+1}.$$ Therefore $P(n)$ holds for all $n \geq 1$.

**Example 1.3.** Compute the sum $3 + 4 + 5 + 6 + \cdots + 1000$.

First compute the sum $1 + 2 + 3 + 4 + 5 + 6 + \cdots + 1000$ using the formula $\frac{n(n+1)}{2}$ for the sum of the integers from 1 through $n$. Then subtract the sum of the integers $1 + 2$ from the above sum.