Discrete Math: Lecture Notes

Roy Joshua
Department of Mathematics, Ohio State University, Columbus, Ohio, 43210, USA.
joshua@math.ohio-state.edu

Abstract
What follows is an informal set of lecture notes.

Contents
1 Lecture 17: 4.4 2
1 Lecture 17: 4.4

Key concepts: Induction II and well-ordering.

Basic idea: Here we want to show that a formula $P(n)$ holds for all integers $n \geq 1$, where $a$ is a fixed positive integer or 0. Induction shows this is true if (i) $P(i)$ is verified to be true for all $a \leq i \leq b$ and (ii) for all $n \geq a$, $P(k)$ for all $b \leq k \leq n$ can be shown to imply $P(n+1)$.

Example 1.1. Suppose $\{c_n|n \geq 0\}$ is a sequence defined by $c_0 = 0$, $c_1 = 1$ and $c_k = 2c_{k-1} - c_{k-1} + 2$ for all $k \geq 2$. Show that $c_n = n^2$ for all $n \geq 0$.

Let $P(n) : c_n = n^2$. Then the LHS of $P(0) = c_0 = 0$. The RHS of $P(0) = 0^2 = 0$. Therefore, $P(0)$ holds.

Next assume $P(k)$ for all $k \leq n$. Then

$$c_{n+1} = 2c_n - c_{n-1} + 2 = 2n^2 - (n-1)^2 + 2 = 2n^2 - n^2 + 2n - 1 + 2 = n^2 + 2n + 1 = (n+1)^2.$$ Therefore, $P(n+1)$ holds.

Well-ordering. This simply says that any nonempty subset of the integers that is bounded below has a least element.

Example 1.2. Show that any positive integer $n$ is divisible by a prime number.

Let $S_n = \{m \in \mathbb{Z} | m > 0, m|n\}$. Then $S$ contains $n$ and therefore is none-empty. It is bounded below by 0 clearly. Therefore there is a least element in $S$, which we call $p$. If $p$ is not prime, then one can factor $p = a.b$, with $0 < a < p$ and $0 < b < p$. Then both $a$ and $b$ would belong to $S$ and this would contradict the fact that $p$ is the smallest element of $S$. Therefore $p$ is prime. Since $p$ belongs to $S$, $p|n$. 

2