Abstract
What follows is an informal set of lecture notes.

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Key concepts: Negation of Universal and existential quantifiers an also statements with several quantifiers

Basic idea: Simple rules: to negate a statement with a universal quantifier - replace the universal quantifier with the existential quantifier and the predicate by its negative. The same applies for the negating a statement with an existential quantifier.

Example 1.1. Exercise 4, p. 95

Write negations of the following: (i) All pots have lids. One may write this as: \( \forall x, \) if \( x \) is a pot, then \( x \) has a lid.

So the negation is: \( \exists x, \) so that \( x \) is a pot and \( x \) does not have a lid. In simple English: Some pots do not have lids.

(ii) Some pigs can fly. One may write this as: \( \exists x, x \) is a pig and \( x \) can fly.

So the negation is: \( \forall x, \) if \( x \) is a pig, then \( x \) cannot fly. More simply: No pigs can fly. Or, all pigs cannot fly.

Next we consider statements with multiple quantifiers. Here one of the key points is that the order of the quantifiers is important.

Example 1.2. Everyone loves somebody.

This becomes: \( \forall x, \exists y \) so that \( x \) loves \( y \). (It is absolutely important here to observe that the \( y \) is allowed to vary depending on \( x \).)

On interchanging the quantifiers we obtain: \( \exists x, \) so that \( \forall y, y \) loves \( x \). i.e. There is someone who is loved by everyone.

\( \exists x, \) so that \( \forall y, x \) loves \( y \). This becomes: There is someone who loves everyone.