Abstract
What follows is an informal set of lecture notes.

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Key concepts: Elementary properties of numbers, prime numbers, even and odd integers, perfect squares etc.

Basic idea: How to prove elementary statements in number theory

Example 1.1. See Exercises 16.

Prove that the sum of any two odd integers is even

Any odd integer can be written as an even integer +1. So if \(a = 2m + 1\) and \(b = 2n + 1\) for some \(m\) and \(n\). Then \(a + b = 2m + 1 + 2n + 1 = 2(m + n) + 2\).

Example 1.2. Is the average of any two odd integers odd? Proceed as before. Then the average of \(a\) and \(b = (a + b)/2 = m + n + 1\). Now \(m + n\) could be either even or odd. For example, if \(m = 1\) and \(n = 3\), \(m + n = 4\) which is even. If \(m = 1\) and \(n = 2\), \(m + n = 3\) which is odd. Therefore, the average could be either odd or even.

Example 1.3. Show that the product of any two odd integers is odd. Let \(a = 2m + 1\), \(b = 2n + 1\). Then \(a.b = (2m + 1)(2n + 1) = 4mn + 2m + 2n + 1 = 2(2mn + n + m) + 1\) which is odd.