A BIFURCATION DIAGRAM FOR OBLIQUE SHOCK INTERACTIONS IN THE UNSTEADY TRANSONIC SMALL DISTURBANCE EQUATION

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Abstract. The UTSD equation is used to model the transition from regular to Mach reflection for weak shocks. Here we examine a related initial value problem: initial data consisting of two oblique shocks in the upper half plane with a symmetry boundary condition on the axis. The slopes, a and $-b$, of the two shocks parameterize the resulting Riemann problem. We identify regions in parameter space corresponding to different qualitative behavior of solutions.

1. Introduction

The unsteady transonic small disturbance (UTSD) equation

$$\begin{aligned} u_t + uu_x + v_y &= 0\,, \\ -v_x + u_y &= 0\,, \end{aligned} \tag{1}$$

has been proposed as a model to study the transition between Mach and regular reflection for weak shocks and small wedge angles. It arises as an asymptotic limit of the Euler equations describing the flow in the neighborhood of the region of interaction of waves. To shed light on the interaction of the incident and reflected shocks, Brio and Hunter, [2], analysed (1) with Riemann data consisting of three states in the upper half plane, separated by shocks. The boundary data on the x-axis are $u_y = 0 = v$. The study in [2] explains the derivation of the model and reports on some numerical simulations. Among their results, Brio and Hunter show that up to an affine change of coordinates one may take

$$\begin{cases} U_0 &= (0,0) \\ U_1 &= (1,-a) \\ U_2 &= (1+a/b,0) \end{cases} \tag{2}$$

for the data, as in Figure 1, which is what we shall use in this paper.

We will refer to the two discontinuities as the "incident" and "reflected" shocks, although in this version of the problem they are given independently. Throughout, $a > 0$, $b > 0$. Though the values of a and b are related to the angle of the wedge and the strength of the shock, Tabak and Rosales have shown, [12], that the shock reflection problem has only a single parameter. Nonetheless, it is interesting to study this Riemann problem in its own right, and the solution to the shock reflection problem could be recovered by tracing a path through a, b-space.

Taking the initial discontinuities to be along $x = ay$ and $x = -by$ results in discontinuities which propagate as shocks. Hence, the basic problem attacked in this paper can be termed the "oblique shock interaction" problem. This contrasts to a study by Zhang and Zheng, [13], which examines only waves parallel to the coordinate axes. Our thesis is that the two-parameter problem

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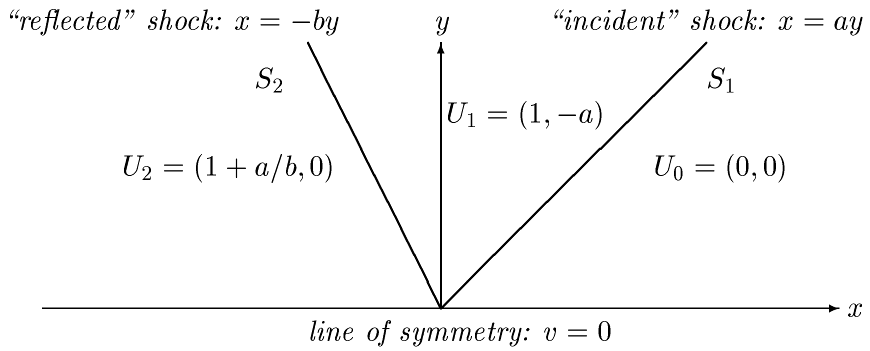

FIGURE 1. Riemann Data for Oblique Shock Reflection Problem

displays qualitative features seen in studies of shock interaction problems — both experimental studies on shock reflection by straight wedges in real gases, theoretical predictions based on shock polar analysis, and numerical studies using models for real and ideal gases, [1], [7], [8], [10], [11]. Shock polar analysis is insufficient, since it assumes constant solutions away from shocks. For weak shocks, small variations in the solutions are no longer negligible. What needs to be substituted is an analysis which takes into account the behavior of the solution in regions where it need not be a constant state or a simple wave; the solution in subsonic regions is particularly important.

The contribution of this paper lies in formulating basic principles for subsonic interactions. We have not yet constructed complete solutions. In this paper we describe how to reduce the self-similar problem to the point where subsonic problems can be identified and studied. In other work, [3], [4], we have begun an analysis of the subsonic problem.

Our results are confined to the simplified model above, for which analysis of the subsonic region seems to be tractable. However, it may apply to the nonlinear wave equation, [6], and the unsteady transonic full-potential equation, [9]. More significant extensions would be necessary to incorporate the effects of linear waves, which are present and influential in the full Euler equations of ideal or real gas dynamics.

Similarity analysis will not give a unique solution for every configuration of data. We believe that all the solutions we discuss are stable to small perturbations of the Riemann data, to perturbation by smoothing of the data, or to viscous perturbation of the equations. This conjecture must also be tested, but this lies beyond the scope of the present paper.

In self-similar coordinates, $\xi = x/t$, $\eta = y/t$, equation (1) becomes

$$\begin{aligned} (u-\xi)u_\xi - \eta u_\eta + v_\eta &= 0\,, \\ -v_\xi + u_\eta &= 0\,. \end{aligned} \tag{3}$$

This can be written as a quasilinear system

$$A(U,\Xi)U_\xi + B(U,\Xi)U_\eta = 0 \tag{4}$$

or in conservation form

$$\partial_\xi F(U,\Xi) + \partial_\eta G(U,\Xi) = S\,, \tag{5}$$

with $U = (u, v)$ and $\Xi = (\xi, \eta)$. Because the coordinate functions appear explicitly in the coefficients of (4) and (5), this system is not standard for hyperbolic conservation law theory. Some basic analysis of shock and rarefaction waves, their construction and admissibility, is carried out in [5]. We summarize the results we need here.

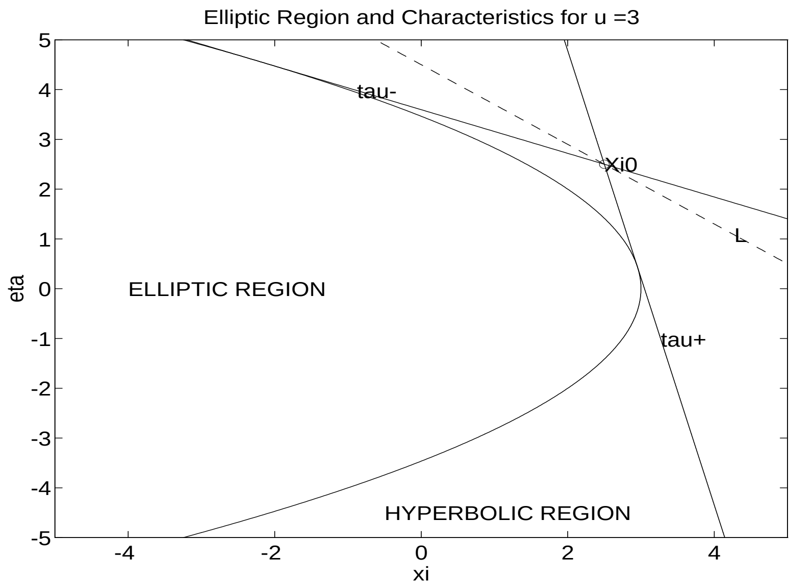

FIGURE 2. The Hyperbolic and Elliptic Regions in (ξ, η)

Equation (3) is of mixed type; it is hyperbolic outside and elliptic inside the parabola

$$P_u : \qquad \xi + \frac{\eta^2}{4} = u.$$

When the equation is linearized about a constant state, the characteristics in the hyperbolic region are straight lines tangent to P_u as sketched in Figure 2.

Shocks between constant states have the form $\xi = \kappa\eta + \omega$ where

$$\kappa \equiv -\frac{[v]}{[u]} = -\frac{v_L - v_R}{u_L - u_R}; \qquad \omega \equiv \kappa^2 + \bar{u} = \kappa^2 + \frac{u_L + u_R}{2}.$$

In the next section we identify the geometry of the elementary figures corresponding to the states in the problem (1), (2).

2. The Geometry of Primary Intersections

We define the following objects.

Shocks: $\mathbf{S_1}$ (between U_0 and U_1): $\xi = a\eta + (1/2 + a^2)$
$\mathbf{S_2}$ (between U_1 and U_2): $\xi = -b\eta + (1 + a/2b + b^2)$
Parabolas: $\mathbf{P_0}$ (corresponding to U_0): $\xi = -\eta^2/4$
$\mathbf{P_1}$ (corresponding to U_1): $\xi = -\eta^2/4 + 1$
$\mathbf{P_2}$ (corresponding to U_2): $\xi = -\eta^2/4 + 1 + a/b$
Axis of Symmetry: $\mathbf{W}$ (wall): $\eta = 0$

These six objects define the *primary curves* of the problem. We study the reduced problem (3) with quasi-one-dimensional data given on the parabolic arc

$$\xi + \eta^2/4 = C \tag{6}$$

for large C. This is consistent with the forward timelike direction in (3) inherited from the original Riemann data for the two-dimensional problem. The data consist of the constant states in (2) with discontinuities at S_1 and S_2.

The *domain of influence* of a point is the union of the compact forward wave cone through the point and the interior of the parabola. For a hyperbolic point (ξ, η) one can also define the *domain*

Intersection	Name	Point
$S_1 \cap S_2$	Ξ_I	$(1/2 + a/2b + ab, 1/2b + b - a)$
$S_2 \cap P_2$	Ξ_0	$(1 + a/2b - b^2 - \sqrt{2ab}, 2b + \sqrt{2a/b})$
	Ξ_*	$(1 + a/2b - b^2 + \sqrt{2ab}, 2b - \sqrt{2a/b})$
$S_1 \cap P_1$	Ξ_1	$(-a^2 + a\sqrt{2} + 1/2, -2a + \sqrt{2})$
$S_1 \cap W$	Ξ_A	$(\omega_1, 0) = (1/2 + a^2, 0)$
$S_2 \cap W$	Ξ_B	$(\omega_2, 0) = (1 + a/2b + b^2, 0)$

TABLE 1. The Primary Intersections

of dependence by following the characteristics of (3) back to (6). For piecewise constant data on (6) there is a region of (ξ, η) space outside of any intersections of the primary curves. We have

Definition 1. *The* primary intersections *are all the intersections of primary curves which could generate new waves. These are $S_1 \cap S_2$, $S_i \cap W$ and $S_i \cap P_i$ for $i = 1$ and 2.*

We name these points in Table 1, for reference.

Remark: It is clear that intersections of one shock with another and of shocks with the wall generate new waves. But the point where a shock intersects the parabola corresponding to one state of the shock may mark a transition from uniform to nonuniform flow. Extending the catalog to include this possibility is the important first step in our approach.

Not all the primary intersections occur in the solution of the Riemann problem; some never occur, and some occur or not depending on the values of a and b. We define those which we expects to occur, and then give a partial classification. Because the characteristics in the hyperbolic region are easily computed, it is straightforward to find the forward domain of influence of the data. However, the solution may contain states of larger magnitude than any of the data at infinity, and hence a computation of the backward domain of dependence of any point is not possible if we do not know the state at that point. We therefore introduce a term for those intersections which are determined *a priori* by the data at infinity, while acknowledging that they may not actually occur.

Definition 2. *An* admissible intersection *is one which appears to be completely determined by the data at infinity.*

We can determine the admissible intersections for each a and b.

Proposition 1. *If Ξ_I is upstream from Ξ_0 then it is admissible.*

Proof: In this case, Ξ_I is in the quasi-one-dimensional domain of dependence of the data at infinity. ∎

Now, Ξ_I lies on P_2 if

$$a^2 + (2b - \frac{3}{b})a + \left(\frac{1}{2b} - b\right)^2 = 0\ .$$

This equation has two solutions $I(a) < K(a)$ for each $a > 0$; $\Xi_I = \Xi_0$ on I, $\Xi_I = \Xi_*$ on K.

Proposition 2. *The shocks meet at the wall if $a = b + 1/2b$.*

Proof: This occurs if $\Xi_I = \Xi_A = \Xi_B$, or $\eta_I = 0$. This gives the equation

$$a = b + \frac{1}{2b} \equiv J(b).$$

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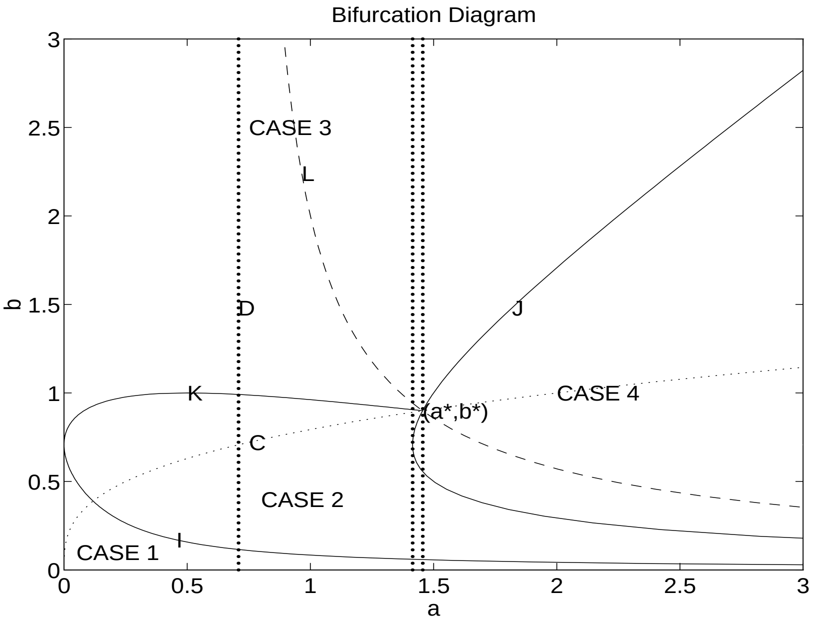

FIGURE 3. Classification of Parameter Space by Position of Ξ_I

In this case the solution of the Riemann problem is a uniform regular reflection. The curve J does not intersect I, but intersects K at a point (a^*, b^*). Since points outside the upper half plane have no meaning in the problem, we consider the curve K to terminate at this point. Classification by position of Ξ_I thus leads to four open regions in the a, b plane:

Case 1: The region below I: Ξ_I above P_2.
Case 2: The region between I and K and left of J: Ξ_I inside P_2, above W.
Case 3: The region above K, left of J: Ξ_I beyond P_2, above W.
Case 4: The region right of J: Ξ_I below W.

See Figure 3, where the curves I, J and K are solid lines.

A classification based on the position of Ξ_I does not determine the qualitative behavior of the flow (for example, Mach or regular reflection), since Ξ_I may not be an admissible point. We note the following elementary results.

Proposition 3. *In Case 1, Ξ_I is the only admissible intersection.*

Proposition 4. *In Cases 2, 3, or 4, Ξ_0 is an admissible intersection.*

Proof: The entire interior of P_2 is downstream from Ξ_0. In Case 2, all the other primary intersections are inside P_2. ■

In Cases 3 and 4 there may be one or two admissible points, depending on whether Ξ_A is inside or outside P_2. We see that Ξ_A is outside P_2 if

$$a > \frac{1}{2b} + \sqrt{\frac{1}{4b^2} + \frac{1}{2}} \equiv L(b).$$

The curve L contains the point (a^*, b^*). We have

Proposition 5. *In either Case 3 or Case 4, if (a, b) lies to the right of $a = L(b)$, there are two admissible primary intersections, Ξ_0 and Ξ_A. If (a, b) lies to the left of this curve, there is only one admissible primary intersection, Ξ_0.*

In Case 1, one or both shocks are nearly vertical, and one can construct a Mach reflection solution by means of quasi-one-dimensional Riemann problems.

3. Solvable Quasi-One-Dimensional Riemann Problems

A quasi-one-dimensional Riemann problem has *data* consisting of a *center* Ξ_0 and two states, U_1, U_2 on a spacelike line, L, through Ξ_0. The *solution* is defined *downstream* (towards the parabola) from L. It is a function of $t \equiv (\xi - \xi_0)/(\eta - \eta_0)$ defined in a forward half-space below L.

A shock on the line $\xi - \xi_0 = t(\eta - \eta_0)$ satisfies, from (5), the *Rankine-Hugoniot relation*

$$F(U, \Xi_0) - F(U_0, \Xi_0) = t(G(U, \Xi_0) - G(U_0, \Xi_0)) \tag{7}$$

where U and U_0 are the constant states on either side of the shock. If (U_0, Ξ_0) is hyperbolic, then eliminating t from (7) defines the *shock polar*; it is a loop, defined for $u \leq u_M(\Xi_0)$; there are two values of v for each $u \neq u_0$, u_M.

Centered simple waves are the other type of local solution $U(t)$. They satisfy

$$(A - tB)\frac{dU}{dt} = 0. \tag{8}$$

Solving (8), we obtain $R(U_0, \Xi_0)$, the *rarefaction polar* connecting to U_0, centered at Ξ_0. Both U and U_0 must be hyperbolic at Ξ_0. For a given point Ξ_0, the curves are defined for $u \leq u_s \equiv \eta_0^2/4 + \xi_0$. Unlike the shock polar, the rarefaction polar does not form a loop, but has two finite and two semi-infinite branches.

Quasi-one-dimensional Riemann problems do not have solutions for all choices of U_1, U_2, and Ξ_0. Nor will the solution always be uniquely defined. The precise result is as follows.

Define the *downstream wave locus*, $D(U_1)$, of a state U_1, with respect to a fixed Ξ_0, as the union of all states which can be joined to U_1 by a shock or rarefaction in which U_1 is the upstream state. The downstream wave locus divides the half-plane $u \leq u_s$ into four regions, labelled 1 – 4 in Figure 4; we label the region above U_1 as '1' and continue counterclockwise. We state

Theorem 1. *Given a Riemann data triple (U_1, U_2, Ξ_0) with hyperbolic states U_1 on the lower half and U_2 on the upper half of a spacelike line L through Ξ_0, let $D(U_1)$ be the downstream wave locus of U_1. Then if U_2 lies in any of the regions 1, 2, or 4 in the complement of D, the Riemann problem has a unique admissible solution. If U_2 lies in region 3, there may be none, one or two admissible solutions.*

Proofs of this and the other results in this section can be found in [5].

Theorem 1 establishes the form of the solution of the hyperbolic Riemann problem which occurs when (a, b) lies below the curve I (Case 1 data).

Theorem 2. *If oblique shock data (2) given for equation (1) are such that $a \leq I(b)$, then a hyperbolic quasi-one-dimensional Riemann problem occurs at Ξ_I. Furthermore, the data for this problem lead to a region 4 configuration, and there is a unique admissible solution, which consists of a rarefaction wave and a shock.*

This result gives a solution to the shock interaction problem for Case 1 data at all supersonic points in the (ξ, η) plane. In [3] and [4] we pursue the subsonic part of the solution. In the first paper, we have solved the degenerate elliptic equation for a fixed boundary, taking account of the fact that a square-root type singularity occurs in the solution at the degenerate boundary. The second paper concerns the free boundary problem which determines the nonuniform part of the Mach stem.

The problem in Case 1 is simplified by the fact that Ξ_I is the only admissible primary intersection, and it organizes the entire problem.

For other cases, the behavior at Ξ_I does not determine the solution; in fact, Ξ_I may not even occur. There are other types of shock interaction where we can formulate quasi-one-dimensional Riemann problems. We have not yet solved the corresponding elliptic (subsonic) problems for these cases.

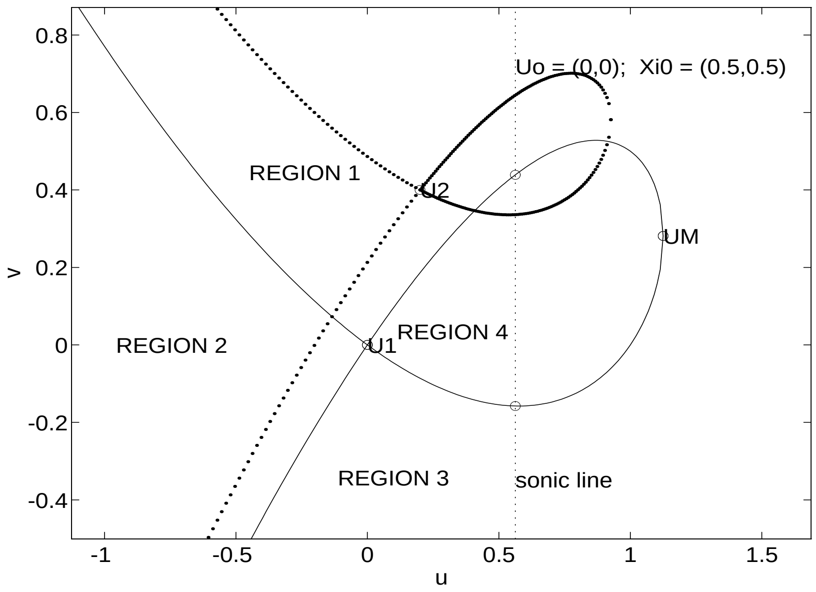

FIGURE 4. Solution of Quasi-One-Dimensional Problems

3.1. **Symmetric Shock Reflection at the Wall.** Proposition 5 shows that Ξ_A is a primary intersection if (a, b) lies to the right of the curve $L(b)$ in Figure 3. The interaction of the incident shock, S_1, with the wall may be reduced to a standard Riemann problem by considering as Riemann data at Ξ_A a spacelike line L (vertical through Ξ_A) and data U_1 on the upper half of the line and the reflected state $U_1^* \equiv (1, a)$ on the lower half.

Proposition 6. *This is a hyperbolic problem if $a \geq 1/\sqrt{2}$.*

For data of this form, if we consider the downstream locus, $D(U_1^*)$, then we are in region 3 of Theorem 1, where we are not guaranteed existence or uniqueness of a solution. In fact, we have

Proposition 7. *The quasi-one-dimensional Riemann problem with the data triple (U_1^*, U_1, Ξ_A) has a solution if and only if $a \geq \sqrt{2}$. If $a > \sqrt{2}$, there are two solutions, $U_R = (1 + a^2 - a\sqrt{a^2 - 2}, 0)$, and $U_F = (1 + a^2 + a\sqrt{a^2 - 2}, 0)$.*

We can formulate the perturbed regular reflection equations for the region of parameter space $\mathcal{R}$ where $a \geq \sqrt{2}$ and $a > I(b)$. Only if $a > L(b)$ is Ξ_A an admissible primary intersection, but it is always *locally admissible*: that is, it lies outside of and upstream from P_1.

For points (a, b) near $J(b)$ there may be solutions close to the respective uniform solutions defined in Proposition 2. We can define two perturbation problems at each point in $\mathcal{R}$, one based on each of the two Riemann solutions of Proposition 7. In both cases, the solution will be piecewise constant outside an elliptic region, in which the solution will satisfy a degenerate elliptic free boundary problem. We conjecture that only for some of these problems will solutions exist; thus, there may be two, one or even no regular reflection type solutions for (a, b) in $\mathcal{R}$.

4. Shock Reflection Problems for Other Parameter Values

4.1. **The UTSD Prototype for von Neumann Reflection.** There is another region of parameter space where we can reduce the problem to solving an elliptic equation. This is the case where $\Xi_1 \equiv S_1 \cap P_1$ is admissible. This requires the two conditions $\eta_1 > 0$ (so that Ξ_1 exists) and $a > I(b)$, so that Ξ_I is not admissible. The first condition, from Table 1, is $a < 1/\sqrt{2}$. We shall call this region $\mathcal{N}$.

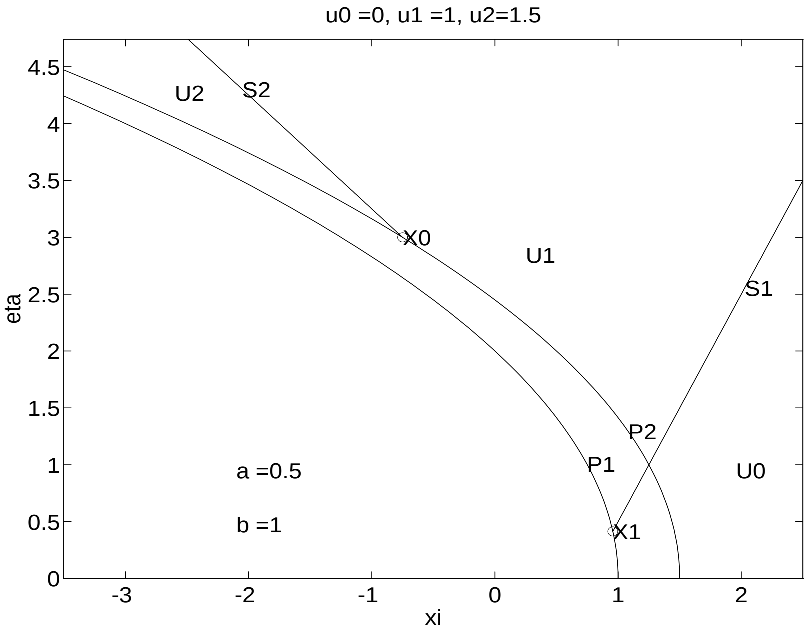

FIGURE 5. The Hyperbolic Solution in von Neumann Reflection

For $(a, b) \in \mathcal{N}$, we again have a reduction of the problem to a degenerate elliptic system. Figure 5 pictures the important variables.

The solution has the constant value U_2 left of the shock S_2 up to the parabola P_2; the shock then becomes nonuniform, decaying and turning until it attains zero strength as it becomes tangent to P_1. The shock S_1 is uniform until it reaches P_1, then it forms a free boundary between a nonuniform, subsonic solution and the upstream state U_0. The elliptic problem in this case has two degenerate parts of the boundary – at P_2 and at P_1 – and two free boundary segments. Details remain to be worked out.

4.2. **A Mechanism for Mach Reflection with a Kink.** Suppose that Ξ_0 is admissible (that is, we are outside Case 1) and that Ξ_A is inside P_2 (that is, $a < L(b)$). In this case, we conjecture that another type of solution may appear: there may be nonuniform *hyperbolic* solutions $(u(\xi, \eta), v(\xi, \eta))$ inside P_2 which approach U_2 at P_2 but with a singularity

$$u \sim -w\sqrt{\rho_2 - \rho}$$

near P_2. Here, $\rho \equiv \eta^2/4 + \xi$ is a parabolic coordinate, and $w > 0$. Singular solutions of the degenerate hyperbolic problem of this form exist, but it is not clear whether the degenerate hyperbolic free boundary problem is well-posed. The free boundary is now the perturbed continuation of S_2, with upstream data the constant state U_1. The Rankine-Hugoniot conditions give sufficient data to determine the nonuniform flow completely. We conjecture that the singular hyperbolic IV-BVP has a unique solution and determines the continuation of S_2 for some distance inside P_2. Then either the continuation of S_2 meets S_1 and a hyperbolic Mach reflection, as in Case 1, occurs or S_2 meets P_1 and disappears, as in von Neumann reflection.

The solution suggested here will look like the prototype Mach reflection but with an extra kink: a bend in the S_2 shock above the intersection point, and a nonuniformity (and singularity) in the solution along P_2. We can describe this prototype as *kinky Mach reflection* (KMR). It shares some features with the phenomenon named 'Transitional Mach Reflection' (TMR) by Ben-Dor, [1].

4.3. **Double Mach Reflection in the UTSD Equation.** If we compare the proposed wave interactions, by region, with theoretical and experimental predictions (see Ben-Dor, [1], for example), there is a rough correspondence between the 'bifurcation diagram' in the $(M_s, \theta_{\mathrm{w}}^C)$ plane and the one that we are building in the (a, b) plane. When the $(M_s, \theta_{\mathrm{w}}^C)$ diagram is rotated 90 degrees counterclockwise and then reflected in a vertical axis, the correspondence becomes apparent. As a qualitative picture of what this model may be capable of showing, namely the relation between different types of two-dimensional wave interactions as parameters vary, the picture so far shows some success: von Neumann reflection and regular reflection are far apart; in fact, they span the parameter space, and are separated by all the other modes of interaction. The major discrepancy is that there is a large region devoted to 'double Mach reflection' in Ben-Dor's experimental and theoretical descriptions, and a big hole in our (a, b) plane, for $1/\sqrt{2} < a < \sqrt{2}$ and b larger than the values which will allow construction of the kinky Mach reflection solution of the preceding section.

Double Mach reflection is difficult to describe in our simplified model. Even the 'simple Mach reflection' of Section 3 (Theorem 2 and the comments following) does not contain a genuine triple point. Mach reflection in this model is in fact a hyperbolic quasi-one-dimensional Riemann problem in which there are four states rather than three and a small rarefaction wave as well as three shocks. There should be a relationship between this simplified Mach stem and that in the full gas dynamics equation model. We have certainly neglected some important features, such as the interaction of linear and nonlinear waves, but we expect to be able to show that the simplified model mimics qualitative features of transitions in real flows.

In our model, a 'double Mach reflection' would have to contain two hyperbolic quasi-one-dimensional problems. We conjecture that one such problem will occur near each of Ξ_0 and Ξ_A in the case when each is admissible but neither gives rise to a hyperbolic Riemann solution ($1/\sqrt{2} < a < \sqrt{2}$ and $a > L(b)$).

In fact, because of the possibility that states U of larger magnitude may appear in the subsonic region, one cannot state *a priori* that any primary interaction is determined by the data at infinity. In particular, the symmetric reflection problem at the wall has no solution, if $a < \sqrt{2}$, irrespective of what other conditions are imposed in the rest of the physical plane. We can find a candidate for a solution by splitting the shock S_1 at a point upstream from Ξ_A and supposing that the shock S_2 splits slightly upstream from Ξ_0. These two triple points, connected by a shock, give a double Mach reflection. As in the other cases we have considered, there is a degenerate elliptic problem which will determine the solution in the subsonic region and will describe what happens to the downstream waves at the second Mach point. The connecting shock is hyperbolic at both end points and transonic for a range in the middle. In this, it resembles the uniform shock which occurs when a lies on the upper branch of $J(b)$.

5. Conclusions

In this paper, we have analysed a simplified model for two-dimensional interactions of weak shocks. We studied a two parameter family of initial data which gives rise to different shock interaction patterns. We showed that the initial value problem for the model equation reduces to solving a quasi-one-dimensional Riemann problem on a curve near infinity. To carry out the analysis, we developed a theory of solutions of quasi-one-dimensional Riemann problems which allows us to construct a solution in the supersonic (hyperbolic) regime. The understanding of a subsonic (elliptic) regime requires solving a series of free boundary problems for a degenerate elliptic equation. When this program is completed, it will provide a full classification of wave interactions, and a precise description of transition criteria between classes of solutions with different qualitative behavior. To the extent of our present knowledge, we present the general bifurcation diagram in Figure 6.

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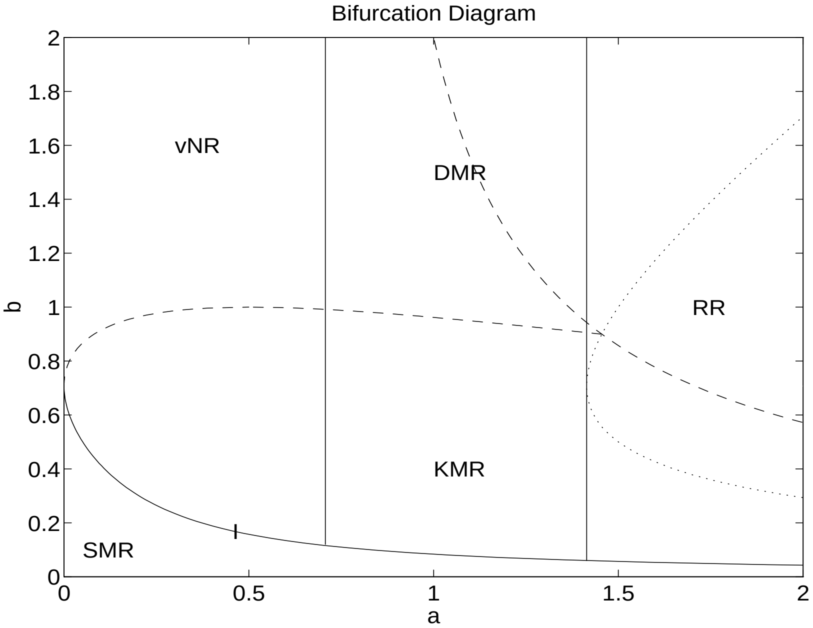

FIGURE 6. Classification of Parameter Space

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