PDE: HOMEWORK 6

Due Friday, October 21st (at the start of the recitation)

- From the Strauss textbook: 1.6.2, 5.2.13, 6.1.13

Date: October 18, 2016.

1.6.2

$-(1+x)y^2 < x^2y^2$ if $y \neq 0$

(since $-(1+x) < x^2$ is satisfied for all x)

Therefore the equation is <u>hyperbolic</u> everywhere except $(x,0)$

$-(1+x)y^2 = x^2y^2$ holds for $y=0$

Therefore, the equation is <u>parabolic</u> for $(x,0)$ except $(-1,0)$ where

$$(1+x) = 2xy = y^2 = 0$$

5.1.13 First we determine the series for e^{ix}

$$c_n = \frac{1}{2l}\int_{-l}^{l} e^{ix} e^{-in\pi x/l}\,dx = \frac{-il}{2l}\,\frac{\exp(ix(l-\pi n)/l)}{l-\pi n}\Bigg|_{x=-l}^{l}$$

$$= -\frac{i}{2}\,\frac{\exp(i(l-\pi n)) - \exp(-i(l-\pi n))}{l-\pi n}$$

Therefore

$$e^{ix} = \sum_{n=-\infty}^{\infty} -\frac{i}{2(l-\pi n)}\left(e^{i(l-\pi n)} - e^{-i(l-\pi n)}\right)e^{in\pi x/l}$$

$$e^{-ix} = \text{——}\,''\,\text{——}\; e^{-in\pi x/l}$$

$$\sin x = \frac{e^{ix}-e^{-ix}}{2i} = -\frac{1}{4}\sum_{n=-\infty}^{\infty}\frac{1}{(\ell-\pi n)}\left(e^{i(\ell-\pi n)}-e^{-i(\ell-\pi n)}\right)\left(e^{in\pi x/\ell}-e^{-in\pi x/\ell}\right)$$

$$= \sum_{n=-\infty}^{\infty}\frac{1}{(\ell-\pi n)}\sin(\ell-\pi n)\sin\left(\frac{n\pi x}{\ell}\right)$$

$$= \sum_{n=1}^{\infty}(-1)^n\left(\frac{1}{\ell-\pi n}-\frac{1}{\ell+\pi n}\right)\sin\ell\,\sin\left(\frac{n\pi x}{\ell}\right)$$

6.1.13 (Following the proof of the max principle for harmonic functions) Let $\varepsilon>0$ and let $v(x)=u(x)+\varepsilon|x|^2$

Then $\Delta v = 4\varepsilon>0$. Therefore, v has no interior maximum. Since v is continuous, it attains the maximum on the boundary. Denote such point on the boundary by x_0. Then for all $x\in D$

$$u(x)\le v(x)\le v(x_0)=u(x_0)+\varepsilon|x_0|^2$$

$$\le \max_{\text{bdy } D} u + \varepsilon\ell^2$$

where ℓ is the greatest distance from bdy D to the origin. Since this is true for any $\varepsilon>0$, we have

$$u(x)\le \max_{\text{bdy } D} u \quad \text{for all } x\in D$$

Thus the maximum of u is attained on the boundary