PDE: QUIZ 1

**Instructions**: No textbooks, notes, devices or materials of any kind are permitted in this quiz. No collaboration is permitted. The quiz will be timed for fifteen (15) minutes. Please submit answers to **all** of the questions.

*All* questions require a written explanation as well as an answer. Your grade is largely depends on showing *correct* and *complete* reasoning. You may assume basic facts about arithmetic or calculus and basics results from class, but please always explain which facts, results or ideas you are using, either by giving their name or by including a precise statement of what they say.

**Academic integrity**: The applicable academic integrity standards and procedures of New York University, the College of Arts and Sciences and the Mathematics Department will be enforced.

**Question 1.** Classify the equations as linear homogeneous, linear inhomogeneous, or nonlinear.

(a) $\tan x \cdot u_{tx} + u_x = \cos(t + x^2)u$
(b) $u_{tt} - e^x u_{txx} = x^2$
(c) $2u_t + 3u_x = \sin u + x^3$
(d) $u_x - u_{yy} = \cos^2(u_x) + \sin^2(u^x)$
(e) $uu_x + u_t = 2u + 3t$

**Question 2.** (a) Write the general solution of

$$\frac{1}{2}u_t - u_x = 0$$

(b) Write the general solution of

$$u_{tt} - 4u_{xx} = 0$$

(c) Use the information from (a) and (b) to find one solution of the nonlinear initial value problem.

$$\begin{cases} u_{tt} - 4u_{xx} = u^3(u_t - 2u_x), t \geqslant 0, x \in \mathbb{R} \\ u(x,0) = e^{-x^2} \end{cases}$$

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1 (a) linear homogeneous

(b) linear nonhomogeneous

(c) nonlinear

(d) linear nonhomogeneous ($u_x - u_{yy} = \cos^2(u_x) + \sin^2(u_x) = 1$)

(e) nonlinear

2 (a) $u(x,t) = g(2t+x)$

(b) $u(x,t) = f(x+2t) + g(x-2t)$

(c) We factor the PDE as

$$\left(\frac{\partial}{\partial t} + 2\frac{\partial}{\partial x}\right)\underbrace{\left(\frac{\partial}{\partial t} - 2\frac{\partial}{\partial x}\right)u}_{v} = u^3\left(\frac{\partial}{\partial t} - 2\frac{\partial}{\partial x}\right)u$$

$$\Updownarrow$$

$$\begin{cases} u_t - 2u_x = v & (1) \\ v_t + 2v_x = u^3 v & (2) \end{cases}$$

$v=0$ trivially solves (2). The solution to $u_t - 2u_x$ is given by $g(2t+x)$ and by the initial condition

$$u(x,t) = g(2t+x) = e^{-(2t+x)^2}$$