WEEK 2

Problem 1 (Missouri 2004 Collegiate Competition)**.** The numbers $\pm 1, \pm 2, ... \pm 2004$ are written on a blackboard. You decide to pick two numbers x and y at random, erase them, and write their product, xy, on the board. You continue this process until only one number remains. Prove that the last number is positive.

Problem 2 (2011 Lehigh Math Contest)**.** List all prime numbers which are of the form $x^3 - 11x^2 - 107x + 1177$ for integer x.

Problem 3. Prove that for $x, y \geqslant 0$, $(x^3 + y^3) \geqslant xy(x + y)$ and $2(x^3 + y^3) \geqslant (x^2 + y^2)(x + y)$.

Problem 4. Prove that for reals a, b, c, $a^2 + b^2 + c^2 \geqslant ab + bc + ac$.

Problem 5 (AM-GM)**.** Prove that for $x, y, z \geqslant 0$, $(x + y + z)/3 \geqslant (xyz)^{\frac{1}{3}}$.

Problem 6 (Schur's inequality)**.** Prove that for $x, y, z, t \geqslant 0$, $x^t(x - y)(x - z) + y^t(y - x)(y - z) + z^t(z - x)(z - y) \geqslant 0$.

Problem 7 (Leo Schneider Student Team Competition)**.** Find equations with integral coefficients whose roots include the numbers

- $\sqrt{2} + \sqrt{3}$
- $\sqrt{2} + \sqrt[3]{3}$

Problem 8. Each point in the plane is assigned a real number such that, for any triangle, the number at the center of its CIRCUMSCIRBED circle is equal to the arithmetic mean of the three numbers at its vertices. Prove that all points in the plane are assigned the same number.

Problem 9 (USAMO 2001)**.** (MUCH harder) Each point in the plane is assigned a real number such that, for any triangle, the number at the center of its INSCRIBED circle is equal to the arithmetic mean of the three numbers at its vertices. Prove that all points in the plane are assigned the same number.

Problem 10 (Putnam 96-A1)**.** Find the least number A such that for any two squares of combined area 1, a rectangle of area A exists such that the two squares can be packed in the rectangle (without interior overlap). You may assume that the sides of the squares are parallel to the sides of the rectangle.

Date: September 2, 2025.