Quiz Sheet 1

Instructions: Turn in responses to TWO questions of your choice. This quiz will be timed for 15 minutes.

Question 1. Show that Green's Theorem is a special case of Stokes' Theorem.

(a) State Green's Theorem.

(b) State Stokes' Theorem.

(c) Express the Green's Theorem line integrals, with $P(x, y)$ and $Q(x, y)$ being the integrands, in the form of the Stokes' Theorem line integral with the integrand $F(P(x, y), Q(x, y), 0)$.

(d) Apply The Stokes Theorem and calculate the surface integral.

Hint: For the surface, consider the positively-oriented region of the xy plane bounded by the integration curve of the line integral. What is the normal vector for such surface?

Question 2. Do all of the following:

(a) State what it means for a subset of $\mathbb{R}^m$ to be *open* in $\mathbb{R}^m$.

(b) State what it means for a map $f : U \to \mathbb{R}^n$ ($U \subseteq \mathbb{R}^m$) to be *continuous at the point* $x_0 \in U$.

(c) State what it means for f to be *continuous on* U.

(d) Suppose $U \subseteq \mathbb{R}^m$ is open. Show that $f : U \to \mathbb{R}^n$ is continuous on U iff for every open set $V \subseteq \mathbb{R}^n$, the set

$$f^{-1}(V) := \{x \in U : f(x) \in V\}$$

is open in $\mathbb{R}^m$.

Question 3. Do all of the following:

(a) State the definition of the *Euclidian metric*.

(b) Suppose $U \subseteq \mathbb{R}^m$ is open. Show that $f : U \to \mathbb{R}^n$ is continuous at $x_0 \in U$ iff for every $\epsilon > 0$, there exists $\delta > 0$ s.t.

$$d(x_0, x) < \delta \Rightarrow d(f(x_0), f(x)) < \epsilon$$

Question 4. Do all of the following:

(a) State the definition of a *limit point* in $\mathbb{R}^m$.

(b) Show that $x_0 := (1, 0, ..., 0)$, where there are $m - 1$ zero entries, is a limit point of the open unit ball

$$\{x \in \mathbb{R}^m : ||x|| < 1\}$$