Quiz Sheet 4

Instructions: Turn in responses to TWO questions of your choice. This quiz will be timed for 15 minutes.

Question 1. Do all of the following. Let $A \subseteq \mathbb{R}^n$.

(a) State what it means for the set *int* A to be the interior, the set *ext* A to be the exterior, and the set ∂A to be the boundary of A.

(b) Suppose $A_1 := B(0,1) = \{x \in \mathbb{R}^n : ||x|| < 1\}$ and $A_2 := \overline{B(0,1)} = \{x \in \mathbb{R}^n : ||x|| \leq 1\}$. Find int A_i, ext A_i, ∂A_i for $i = 1, 2$.

Question 2. Prove that $x \in \partial A \subseteq \mathbb{R}^n$ if and only if every open set containing x intersects both A and $\mathbb{R}^n \setminus A$.

Question 3. In "Piece One" (invertibility of $\varphi'(a)$ implies φ is injective near a) of the proof of the Inverse Function Theorem we established the following. Let U be open in $\mathbb{R}^n$ and $\varphi : U \to \mathbb{R}^n$ be of class C^1. If $\varphi'(a)$ is invertible then there exists $\alpha > 0$ s.t.

$$||\varphi(x_0) - \varphi(x_1)|| \geq \alpha||x_0 - x_1|| \ (*)$$

for all $x_0, x_1 \in B(a, \epsilon)$ and some $\epsilon > 0$. Explain why local injectivity of φ follows from $(*)$.

Question 4. Give an example of $\varphi : U \to \mathbb{R}^n$ that is continuous on U and $B(a, \epsilon) \subseteq U$ open such that $\varphi(B(a, \epsilon)) \subseteq \mathbb{R}^n$ is NOT open, or explain why such result may occur.