

MATH 3345: IN-CLASS EXERCISES FOR 1/11/2019 AND 1/14/2019

1. What is the negation of a conditional? Express $\neg(P \Rightarrow Q)$ in a different way, without using the connective \Rightarrow .

2. (Exercise 9 on page 11 in textbook) Let P xor Q denote “exclusive or”, so that P xor Q is true when either P is true or Q is true (but not both). Write out the truth table for this connective, and show that the following are logically equivalent:

$$P \text{ xor } Q, \quad (P \wedge \neg Q) \vee (Q \wedge \neg P), \quad \neg(P \Leftrightarrow Q), \quad (\neg P) \Leftrightarrow Q, \quad P \Leftrightarrow (\neg Q).$$

3. Let P , Q , and R be sentences. We proved the distributive law $P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$ in class. This is Proposition 2.4b in the textbook; in Exercise 2 of the homework, you will prove Proposition 2.4a, namely that

$$P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$$

both by writing out a truth table and by an explanation in words. Can you give a more efficient proof of this distributive law?

4. Construct the truth tables for the following statements, and for each one determine whether it is a tautology, a contradiction, or neither. How would you answer this question without using truth tables?

- (a) $\neg(P \wedge Q) \Rightarrow (\neg P \wedge \neg Q)$
- (b) $(\neg P \wedge \neg Q) \Rightarrow \neg(P \wedge Q)$
- (c) $[(P \vee Q) \wedge R] \Leftrightarrow [P \vee (Q \wedge R)]$
- (d) $(P \Leftrightarrow Q) \Leftrightarrow [(P \wedge Q) \vee \neg(P \vee Q)]$

5. For each of the following, replace the symbol $*$ with a connective so that the resulting symbolic statement is a tautology. (If you can, figure these out without using truth tables.)

- (a) $[(P \Rightarrow (Q * R))] \Leftrightarrow [(P \Rightarrow Q) \wedge (P \Rightarrow R)]$
- (b) $[(P * Q) \Rightarrow R] \Leftrightarrow [(P \Rightarrow R) \wedge (Q \Rightarrow R)]$
- (c) $[(P \wedge Q) \Leftrightarrow P] \Leftrightarrow [P * Q]$
- (d) $[(P * Q) \Rightarrow R] \Leftrightarrow [P \Rightarrow (Q \Rightarrow R)]$