Last time:

③ What are your goals for MA 261?

- presentation + public speaking skills
- use LaTeX
- flexibility in problem solving
- problem solving skills
- learn some interesting math
- clarify past uncertainty about concepts

④ What do you expect from me?

- Help navigating proofs
- open channel of communication
- Consistency

Intro: Name, favorite TV show/movie

⑤ What is the value of making mistakes in the learning process?

- Learn what not to do
- Freedom to learn
- Productive failure / Productive struggle

6. How do we create an environment in which risk-taking is encouraged and making mistakes is okay?

- Connect with one another
- Don't punish mistakes

Introduction to proofs

Definitions

In order to precisely state and prove theorems, we first must agree on clear **definitions** of the mathematical objects and concepts involved.

Here are two definitions of the concept of evenness:

Definition 1: An integer n is **even** if it is in the set $\{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}$.

Definition 2: An integer n is **even** if there is another integer k such that $n = 2k$.

Which of these definitions do you prefer? Why?

Def 2 - more precise, gives a recipe

Def 1 - more concrete, gives examples

Definition 2 might be rephrased as: "a number is even if it is divisible by 2." Let's make this more precise.

Definition 3: Let n and d be two integers. We say that d **divides** n, or equivalently that n **is divisible by** d if there is another integer k such that $n = dk$. In this case, we write $d \mid n$.

Mathematicians often use examples to illustrate a definition.

Example 1: The number 3 divides 12 because $12 = 3 \cdot 4$. This also shows that $4 \mid 12$. We can see that $2 \mid 12$ and $6 \mid 12$ from the expression $12 = 2 \cdot 6$. Finally, $1 \mid 12$ and $12 \mid 12$ because $12 = 1 \cdot 12$.

We can therefore conclude that

$$1, 2, 3, 4, 6, 12$$

is a complete list of positive integers which divide 12.

Example 2: If n is any integer, then $1 \mid n$ and $n \mid n$. (Why?) $n = 1 \cdot n$

Definition 4: Let $p > 1$ be an integer. We say that p is **prime** if 1 and p are the only positive integers which divide p.