$a=7$

$b=1$

$c=5$

$n=2$

<u>Hypothesis</u>: $a \equiv b \pmod{n}$ $\qquad$ $b \equiv c \pmod{n}$

$\Updownarrow$ $\qquad$ $\Updownarrow$

$n \mid (a-b)$ $\qquad$ $n \mid (b-c)$

<u>Goal</u>: $a \equiv c \pmod{n} \iff n \mid (a-c)$

<u>Key step</u>: write $a = k_1 n$

$b = k_2 n$

$c = k_3 n$

<u>Example</u>: $n=2$

$a = 2 \cdot 2$

$b = 5 \cdot 2$

$c = 50 \cdot 2$

$a=4$, $b=10$, $c=100$

$4 \equiv 10 \pmod{2}$ $\qquad$ $10 \equiv 100 \pmod{2} \rightarrow 4 \equiv 100 \pmod{2}$ ✓

$a=7$, $b=1$, $c=5$

$7 \equiv 1 \pmod{2}$, $\qquad$ $1 \equiv 5 \pmod{2} \rightarrow 7 \equiv 5 \pmod{2}$ ✓

$7-1=6$
$= 2 \cdot \underline{3}$

$1-5=-4$
$= 2 \cdot \underline{(-2)}$

$7-5=2$
$= 2 \cdot \underline{1}$

$a \equiv b \pmod{n}$ $\Updownarrow$ $n \mid (a-b)$ $\Updownarrow$ $\underline{a - b} = n \cdot k_1$

$b \equiv c \pmod{n}$ $\Updownarrow$ $n \mid (b-c)$ $\Updownarrow$ $\underline{b-c} = n \cdot k_2$

<u>Goal</u>

$a \equiv c \pmod{n}$ $\Updownarrow$ $\underline{n \mid (a-c)}$

$$\begin{array}{rl} (a - \cancel{b}) & = n \cdot k_1 \\ + \; (\cancel{b} - c) & = n \cdot k_2 \\ \hline \underline{a - c} & \underline{= nk_1 + nk_2 = n(k_1 + k_2)} \end{array}$$

✓

Michael + Ashe just proved:

If $n > 0$ and a, b, c are any integers,

① $a \equiv a \pmod{n}$ [congruence is "reflexive"]

② If $a \equiv b \pmod{n}$, then $b \equiv a \pmod{n}$ [congruence is "symmetric"]

③ If $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$,

then $a \equiv c \pmod{n}$ [congruence is "transitive"]

Together, these 3 properties mean congruence modulo n is an <u>equivalence relation</u>.

<u>Other examples</u>: $=$, similar triangles