UK Math Club Spring Social

Tomorrow, 2/11, 5 PM

Zoom ID: 444 869 5740

Password: math

Graded HW back this afternoon

✓+ (1/1), ✓ (.8/1), ✓- (.5/1)

Exam 1 next Wednesday, 2/17

- divisibility
- congruence mod n
- proof techniques, logic, style

Proofs (1 old, 1 new)
Examples
Proof critique

a, b, n integers, $n > 0$.

<u>1.15</u>: If $a \equiv b \pmod{n}$, then $a^2 \equiv b^2 \pmod{n}$

<u>1.16</u>: If $a \equiv b \pmod{n}$, then $a^3 \equiv b^3 \pmod{n}$

<u>1.17</u>: If $a \equiv b \pmod{n}$ $\boxed{\text{AND } a^{k-1} \equiv b^{k-1} \pmod{n}}$ then $a^k \equiv b^k \pmod{n}$

Is this a necessary hypothesis?

Get this infinite string of theorems:

If $a \equiv b \pmod{n}$ then $a^2 \equiv b^2 \pmod{n}$

then $a^3 \equiv b^3 \pmod{n}$

then $a^4 \equiv b^4 \pmod{n}$

⋮

Thm 1.18: Let a, b, k, n be integers, $n > 0$, $k > 0$.

If $a \equiv b \pmod{n}$, then $a^k \equiv b^k \pmod{n}$

This is 1.17 with the hypothesis $a^{k-1} \equiv b^{k-1} \pmod{n}$ removed!

"Proof": Start w/ $a \equiv b \pmod{n}$ + multiply LHS by a
RHS by b

again and again and again ...

Formalize with mathematical induction.

Our theorem is really a bunch of theorems, one for each value of k.

2 pieces to the proof:

Knock over the first domino: Show $a \equiv b \pmod{n} \Rightarrow a^2 \equiv b^2 \pmod{n}$

Thm 1.15

If $(k-1)$st domino goes over, it will knock over kth domino: Show $a \equiv b \pmod{n}$, $a^{k-1} \equiv b^{k-1} \pmod{n}$ $\Rightarrow a^k \equiv b^k \pmod{n}$

Thm 1.17

Conclude: Every domino gets knocked over, i.e.

$a \equiv b \pmod{n} \Rightarrow a^k \equiv b^k \pmod{n}$

is true for every $k > 0$.