Our proofs can, and should, use previous results.

Ex: Thm 1.18: a, b, n, k integers, $n > 0$, $k > 0$.

If $a \equiv b \pmod{n}$, then $a^k \equiv b^k \pmod{n}$

$$10 \equiv 1 \pmod{3} \underset{\text{Thm 1.18}}{\Longrightarrow} 10^k \equiv 1^k \pmod{3}$$
$$\equiv 1 \pmod{3}.$$

$(10 - 1 = 9 = 3 \cdot 3)$

Ex: $2,704 \equiv 2 + 7 + 0 + 4 \pmod{3}$
$\equiv 13 \pmod{3}$
$\equiv 1 \pmod{3}$

$\Rightarrow$ 2704 is not divisible by 3.

1.24: Come up with other similar divisibility criteria.

1.22/1.23: • If n divisible by 3, then the sum of its digits is divisible by 3

• If the sum of n's digits is divisible by 3, then n is divisible by 3

Condensed version:

Thm: A natural number in base 10 is divisible by 3 if and only if the sum of its digits is div. by 3.

↳ 2 things to prove

Axioms

Axiom 1: There exists a set of integers

$$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

with operations addition $(+)$ and multiplication $(\cdot)$ satisfying

- the sum (or product) of two integers is another integer.
- Commutativity: $a+b = b+a$
  $ab = ba$
- Associativity: $(a+b)+c = a+(b+c)$
  $(ab)c = a(bc)$
- Distributive Property: $a(b+c) = ab + ac$

- <u>Identity elements</u>: $a + 0 = a$

  $a \cdot 1 = a$

- <u>Negatives</u>: For each integer a, there is an integer $-a$ such that

$$a + (-a) = 0.$$

<u>Axiom 2</u> (Well-ordering of the natural numbers)

The natural numbers are

$$\mathbb{N} = \{1, 2, 3, \dots\}.$$

If S is any non-empty subset of $\mathbb{N}$, then S has a smallest element.

<u>Thm</u>: Every natural number is <u>interesting</u>.

<u>Proof</u>: If not, then there are some uninteresting numbers. Let S be the set of all uninteresting numbers.

Let n be the smallest uninteresting number.

Then n is interesting, because it's the smallest uninteresting number, which is a contradiction. $\blacksquare$