1.31 : #1 $(36, 22)$

$$36 = \cancel{n_1}^{\,1} \cdot 22 + \cancel{r_1}^{\,14}$$

$$(36, 22) = (22, 14)$$

$$36 = 1 \cdot \underline{22} + \underline{14}$$

$$22 = \cancel{n_2}^{\,1} \cdot 14 + \cancel{r_2}^{\,8}$$

$$(22, 14) = (14, 8)$$

$$22 = 1 \cdot \underline{14} + \underline{8}$$

$$14 = 1 \cdot \underline{8} + \underline{6}$$

$$(14, 8) = (8, 6)$$

$$8 = 1 \cdot \underline{6} + \underline{2}$$

$$(8, 6) = (6, 2)$$

$$6 = 3 \cdot 2 + 0$$

$$(6, 2) = 2$$

<u>Euclidean algorithm</u>

<u>For Wednesday</u>: No daily HW
Start your first portfolio entries.

<u>For Friday</u>: Induction worksheet

<u>Thm I.1</u>: Let n be a natural number. Then

$$1+2+3+\cdots+n = \frac{n(n+1)}{2}$$

<u>Proof</u>: We'll prove this by induction on n.

<u>Base Case</u>: When $n=1$, the left hand side is

$$1$$

and the right hand side is

$$\frac{1(1+1)}{2} = \frac{2}{2} = 1. \quad \checkmark$$

So the theorem is true for $n=1$.

<u>Inductive Step</u>: Let k be a natural number and

Inductive Hypothesis

assume the theorem is true when $n=k$.

That is, assume

$$1+2+3+\cdots+k = \frac{k(k+1)}{2}.$$

Now, let's show the theorem will be true when $n=k+1$.

$$(1+2+3+\cdots+k)+(k+1) = \frac{k(k+1)}{2} + (k+1)$$

by the inductive hypothesis.

Now, simplify:

$$1+2+3+\cdots+k+(k+1) = \frac{k(k+1)}{2} + \frac{2(k+1)}{2}$$

$$= \frac{k^2+k+2k+2}{2}$$

$$= \frac{k^2+3k+2}{2}$$

$$= \frac{(k+2)(k+1)}{2}$$

$$= \frac{((k+1)+1)(k+1)}{2}.$$

This shows that the theorem is true when $n=k+1$.

Conclude that, by induction, the theorem is true for every nat. number n. ∎