Exams graded

Next round of presentations assigned

- Friday 2/26 – Monday 3/15
- Put it on your calendar
- If you're giving presentation #2, think about what (if anything) you want to change.

Portfolio problems assigned

- First draft due by Monday, 3/8.

Thm I.2: For every natural number n, $\{1, 2, 3, 4, \dots\}$

$$\underbrace{1 + 2 + 2^2 + 2^3 + \cdots + 2^n}_{= \sum_{i=0}^{n} 2^i} = 2^{n+1} - 1$$

Proof: Base case: $n = 0$

In this case, when $n=0$, the theorem states

$$2^0 = 2^{0+1} - 1$$

i.e., $1 = 1$, which is true.

Inductive step: Assume the theorem is true for some integer $n \geq 0$.

Inductive Hypothesis

That is,

$$1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1.$$

Now,

$$\left(1 + 2 + 2^2 + \cdots + 2^n\right) + 2^{n+1}$$

$$= \left(2^{n+1} - 1\right) + 2^{n+1}$$

$$= 2^{n+1} + 2^{n+1} - 1$$

$$= 2 \cdot 2^{n+1} - 1$$

$$= 2^{n+2} - 1$$

$$= 2^{(n+1)+1} - 1.$$

This completes the inductive step.

We conclude the theorem is true for all $n \geq 0$. ∎

Let $P(n)$ be the statement

$$1 + 2 + 2^2 + 2^3 + \cdots + 2^n = 2^{n+1} - 1$$

- $P(0)$ is the statement $1 = 2^1 - 1$, which was our base case.
- In the inductive step, we assume $P(n)$ is true

$$1 + 2 + 2^2 + 2^3 + \cdots + 2^n = 2^{n+1} - 1$$

given equality

and we use it to prove $P(n+1)$ is true

$$1 + 2 + 2^2 + 2^3 + \cdots + 2^n + 2^{n+1} = 2^{(n+1)+1} - 1$$

Not given. We need to prove it.

Friday: I.3 - I.7

Base case ✓

Outline the inductive step

- What is $P(n)$, which we're assuming to be true?
- What is $P(n+1)$, which we want to prove?
- Now try to prove $P(n+1)$ using $P(n)$.