Thm: For every natural number $n \geq 7$,

$$3^n < n!$$

n	3^n	$n!$
1	3	1
2	9	2
3	27	6
4	81	24
5	243	120
6	729	720
7	2187	5040

Proof: Base Case: $n=7$

$$3^7 = 2187 < 5040 = 7!$$

Induction step: Suppose that

$$\boxed{3^n < n!}$$

Inductive hypothesis

for some number $n \geq 7$

We want: $3^{(n+1)} < (n+1)!$

$$(n+1)! = (n+1)\,n!$$

$$= n \cdot n! + n!$$

$> n \cdot 3^n + 3^n$

$> 1 \cdot 3^n + 3^n$

$= \cancel{2}\, 3^n$

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Since $n \geq 7 > 2$ and $n! > 3^n$ by assumption,

$$(n+1)! > 2 \cdot 3^n + 3^n = 3 \cdot 3^n = 3^{(n+1)}.$$

Thus, the theorem holds for $n+1$.

By induction, it holds for all integers $n \geq 7$. ∎

$$(n+1)! = (n+1)\cdot n! = n\cdot n! + n!$$
$$> 2\cdot n! + n!$$
$$= 3\cdot n!$$
$$> 3\cdot 3^n$$
$$= 3^{n+1}$$

$n \geq 7 > 2$

$\boxed{3^n < n!}$

$$\boxed{(1+2+3+\dots+n)^2} = \left(\frac{n(n+1)}{2}\right)^2 = \frac{n^2(n+1)^2}{4}$$

$$\boxed{1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}}$$

$$(1+2+3+4)^2 = 1^3 + 2^3 + 3^3 + 4^3$$