$$\left(\frac{3}{2}\right)^{n-2} \leq F_n < \left(\frac{5}{3}\right)^n$$

1, 1, 2, 3, 5, 8, 13, ...

What is $\lim_{n \to \infty} \frac{F_{n+1}}{F_n}$?

Golden Ratio ≈ 1.61803

$$\boxed{a = \frac{1+\sqrt{5}}{2}}, \quad b = \frac{1-\sqrt{5}}{2} = -\frac{1}{a}$$

$$F_n = \frac{a^n - b^n}{a - b}$$

$$\frac{F_{n+1}}{F_n} = \frac{a^{n+1} - b^{n+1}}{a^n - b^n}$$

- HW backlog
  - expect a few graded HW emails
- First portfolio entry
  - share by Monday

Recall: Euclidean Algorithm for finding the greatest common divisor.

Ex: $(248, 72)$

$$248 = 3 \cdot 72 + 32 \qquad 0 \le r \le 71$$

$$3 \cdot 72 = 216$$

$$72 = 2 \cdot 32 + \boxed{8}$$

GCD

$$2 \cdot 32 = 64$$

$$32 = 4 \cdot 8 + 0$$

By Thm 1.33: $(248, 72) = (72, 32)$

$= (32, 8)$

$= (8, 0)$

$= 8$

Big Theorem for next time

Thm 1.38/1.39: Let a and b be integers. Then a and b are relatively prime ($(a,b)=1$) if and only if there exist integers x and y with

$$ax + by = 1.$$

2 things to prove

I'll present this on Friday → 1.38: $(a,b)=1 \implies$ there exist x and y with $ax+by=1$

HW for Friday → 1.39: there exist x and y with $ax+by=1 \implies (a,b)=1$

Ex: Let $a=18$, $b=7$. Need to "solve"

$$18x + 7y = \boxed{1}$$

$$18 = 2\cdot 7 + \boxed{4} \rightarrow 4 = 18 - 2\cdot 7$$

$$7 = 1\cdot 4 + \boxed{3} \rightarrow 3 = 7 - 1\cdot \underline{4}$$

$$4 = 1\cdot 3 + \boxed{1} \rightarrow 1 = 4 - 1\cdot \underline{3}$$

$$3 = 3\cdot 1 + 0$$

$$\begin{aligned} 1 &= 4 - 1\cdot \underline{3} \\ &= 4 - (7 - 1\cdot 4) = 2\cdot \underline{4} - 7 \\ &= 2(18 - 2\cdot 7) - 7 \\ &= 2\cdot 18 - 5\cdot 7 \end{aligned}$$

So

$$1 = 18x + 7y$$

is solved by $x = 2$, $y = -5$.