Ex: $a=4, \quad b=3$

$$\underbrace{\gcd(4,3)}_{=1} \cdot \underbrace{\operatorname{lcm}(4,3)}_{=12} = 4 \cdot 3 = 12$$

Ex: $a=4, \quad b=6$

$$\underbrace{\gcd(4,6)}_{=2} \cdot \underbrace{\operatorname{lcm}(4,6)}_{=12} = 4 \cdot 6 = 24$$

$$\gcd(a,b) \cdot \operatorname{lcm}(a,b) = ab \quad \rightarrow \quad \operatorname{lcm}(a,b) = \frac{ab}{\gcd(a,b)}$$

$$\gcd(a,b) = \frac{ab}{\operatorname{lcm}(a,b)}$$

- All HW through last week returned by tomorrow.
- Feedback on portfolio revisions by tomorrow.
- Usual office hour today at 3 pm.

Exam 2 - Wednesday

- Exam available on Canvas as a pdf at 11 AM
- Upload solutions as pdf by 12 pm

- Max in Zoom classroom to answer questions
- 5 problems
  - some computation / examples
  - proofs
    - ↳ at least one by induction
    - ↳ at least one from the HW
    - ↳ at least one that's new

- Content:
  - Division Algorithm

    Given $n, m \in \mathbb{Z}$, there exist unique $q, r \in \mathbb{Z}$ with

    $$m = qn + r \longleftrightarrow \frac{m}{n} = q + \frac{r}{n}$$

    and $0 \leq r \leq n-1$

  - GCDs
  - Euclidean Algorithm

    Repeated division algorithm.

    $$a = q_1 b + r_1$$

    $$b = q_2 r_1 + r_2$$

    $$\vdots$$

    Last nonzero remainder is $\gcd(a,b)$.

  - Induction
  - Linear Diophantine Equations

Executive Summary of LDEs

$$ax + by = c \qquad a, b, c \in \mathbb{Z}$$

- This has a solution (integers x, y) if and only if (a,b) divides c. Thm 1.48

- If there are solutions, we can find one by
  - guessing + checking
  - reverse engineer the Euclidean alg.

$$
\begin{aligned}
a &= q_1 b + r_1 \\
b &= q_2 r_1 + r_2 \\
r_1 &= q_3 r_2 + r_3 \\
&\vdots \\
r_{k-1} &= q_{k+1} r_k + (a,b) \quad \to \quad (a,b) = r_{k-1} - q_{k+1} r_k
\end{aligned}
$$

solve for remainder in each eq.

- Once you have one solution, call it x_0, y_0, All other solutions are

$$x = x_0 + k \cdot \frac{b}{(a,b)}$$

$$y = y_0 - k \cdot \frac{a}{(a,b)}$$

where $k \in \mathbb{Z}$. Thm 1.53.

Ex: $12x + 33y = 15$

Thm 1.40: We have solutions $\Leftrightarrow$ $(12,33)$ divides 15.

Euclidean Alg: $33 = 2 \cdot 12 + 9$

$12 = 1 \cdot 9 + \boxed{3} \rightarrow (12,33) = 3$

$9 = 3 \cdot 3$

Since $3 \mid 15$, there will be solutions.

To find one solution: Reverse the Euclidean Alg:

$33 = 2 \cdot 12 + 9 \rightarrow 9 = 33 - 2 \cdot 12$

$12 = 1 \cdot 9 + 3 \rightarrow 3 = 12 - 9$

So

$$\begin{aligned} 3 &= 12 - 9 \\ &= 12 - (33 - 2 \cdot 12) = 12 - 33 + 2 \cdot 12 \\ &= 3 \cdot 12 - 33 \end{aligned}$$

$$5\cdot\left(12(3)+33(-1)\right)=(3)\cdot 5$$

$$12(15)+33(-5)=15$$

$180 \qquad -165$

Find All solutions:

$$12x+33y=15$$

One sol. is $x_0=15$, $y_0=-5$.

Other solutions:

$$x=15+k\cdot\frac{33}{(12,33)}, \qquad y=-5-k\cdot\frac{12}{(12,33)}$$

$$=15+k\cdot 11 \qquad\qquad =-5-k\cdot 4$$

where k is any integer

x	y	⋮
-18	7	k = -3
-7	3	k = -2
4	-1	k = -1
15	-5	k = 0
26	-9	k = 1
37	-13	k = 2
48	-17	⋮