- iPad notes now available on HW page
- Office hours survey - respond today if you want input
- Next presentations - respond ASAP
- Exam 2 graded - 20 point curve

Thm 2.1: McKenzie's proof can be understood as a strong induction:

Assume every k with $1 < k < n$ has a prime divisor.

Case 1: n is prime ✓

Case 2: n is composite $\Rightarrow n = xy$,

where $1 < x, y < n$

By inductive hyp., some $p \mid x \Rightarrow p \mid n$.

Thm 2.3: $n > 1$ is prime if and only if every prime $p \leq \sqrt{n}$ does not divide n.

n is prime $\Longrightarrow$ every prime $p \leq \sqrt{n}$ doesn't divide n ✓

$\boxed{\text{every prime } p \le \sqrt{n} \text{ doesn't divide } n \implies n \text{ prime}}$

Equivalently,

negate negate

n NOT prime $\implies$ some prime $p \le \sqrt{n}$ does divide n

Ex: 101 is prime $\sqrt{101} \approx \sqrt{100} = 10$

Check 2, 3, 5, 7 don't divide 101

47 $\sqrt{47} < \sqrt{49} = 7$

$2 \nmid 47$, $3 \nmid 47$, $5 \nmid 47$ $\implies$ 47 is prime.

Next HW: 2.7 – 2.9

Fundamental Theorem of Arithmetic:

Every natural number $n > 1$ can be written uniquely as a product of primes.

↳ up to reordering

Ex: $72 = 12 \cdot 6$

$$= (4 \cdot 3)(2 \cdot 3)$$

$$= (2 \cdot 2 \cdot 3)(2 \cdot 3) = 2^3 \cdot 3^2$$

Existence (2.7) - use 2.1 and strong induction

Uniqueness (2.9) - Let

$$\{p_1, \dots, p_m\}$$

$$\{q_1, \dots, q_s\}$$

be sets of primes with $\left(\begin{matrix} p_i \neq p_j \\ q_i \neq q_j \end{matrix} \quad \text{if } i \neq j \right)$

no repeats

If

$$n = p_1^{r_1} \cdots p_m^{r_m}$$

$$= q_1^{t_1} \cdots q_s^{t_s}$$

then these are actually the same factorization

- $m = s$
- $\{p_1, \dots, p_m\} = \{q_1, \dots, q_s\}$
- if $p_i = q_j$, then $r_i = t_j$

Ex: $72 = \underline{2^3 \cdot 3^2}$ $\quad p_1 = 2, \quad p_2 = 3$

$= \underline{3^2 \cdot 2^3}$ $\quad q_1 = 3, \quad q_2 = 2$

Idea: Lemma 2.8 + induction

No class Friday