Thm 2.27: Let p be a prime and a, b integers. If $p \mid ab$, then $p \mid a$ **or** $p \mid b$.

non-exclusive

Proof: Let

$$a = p_1^{r_1} \cdots p_m^{r_m}$$

and

$$b = q_1^{t_1} \cdots q_s^{t_s}$$

be the prime factorizations of a and b. Then

$$ab = (p_1^{r_1} \cdots p_m^{r_m})(q_1^{t_1} \cdots q_s^{t_s}).$$

Since $p \mid ab$, so by Lemma 2.8 p must equal one of the primes in this product. That is,

$$p = p_i \text{ for some } i \Rightarrow p \mid a$$

non-exclusive or

$$p = q_j \text{ for some } j \Rightarrow p \mid b.$$

∎

Ex: $3 \mid 6 \cdot 12 \Rightarrow 3 \mid 6$ and $3 \mid 12$

$3 \mid 5 \cdot 12 \Rightarrow 3 \nmid 5$ but $3 \mid 12$

Non-Ex: $9 \mid 6 \cdot 12$ but $9 \nmid 6$ and $9 \nmid 12$.

$6 \cdot 12 = 72 = 8 \cdot 9$

Portfolio assignments - today

Exam 3 - Next Wednesday, 4/14

(Chapter 2 test)

- Primes
- FTA
- Applications of FTA (more properties of primes, irrationality, gcds and lcms)

Next HW: 2.28 - 2.31