2.22: $24x \equiv 123 \pmod{213}$

$\iff 24x + 213y = 123$ (2.19)

One solution: $x_0 = 369$, $y_0 = -41$

Thm 1.53: If x_0, y_0 is one sol. to $ax+by=c$, then all solutions are

$$x = x_0 + k \cdot \frac{b}{(a,b)}$$

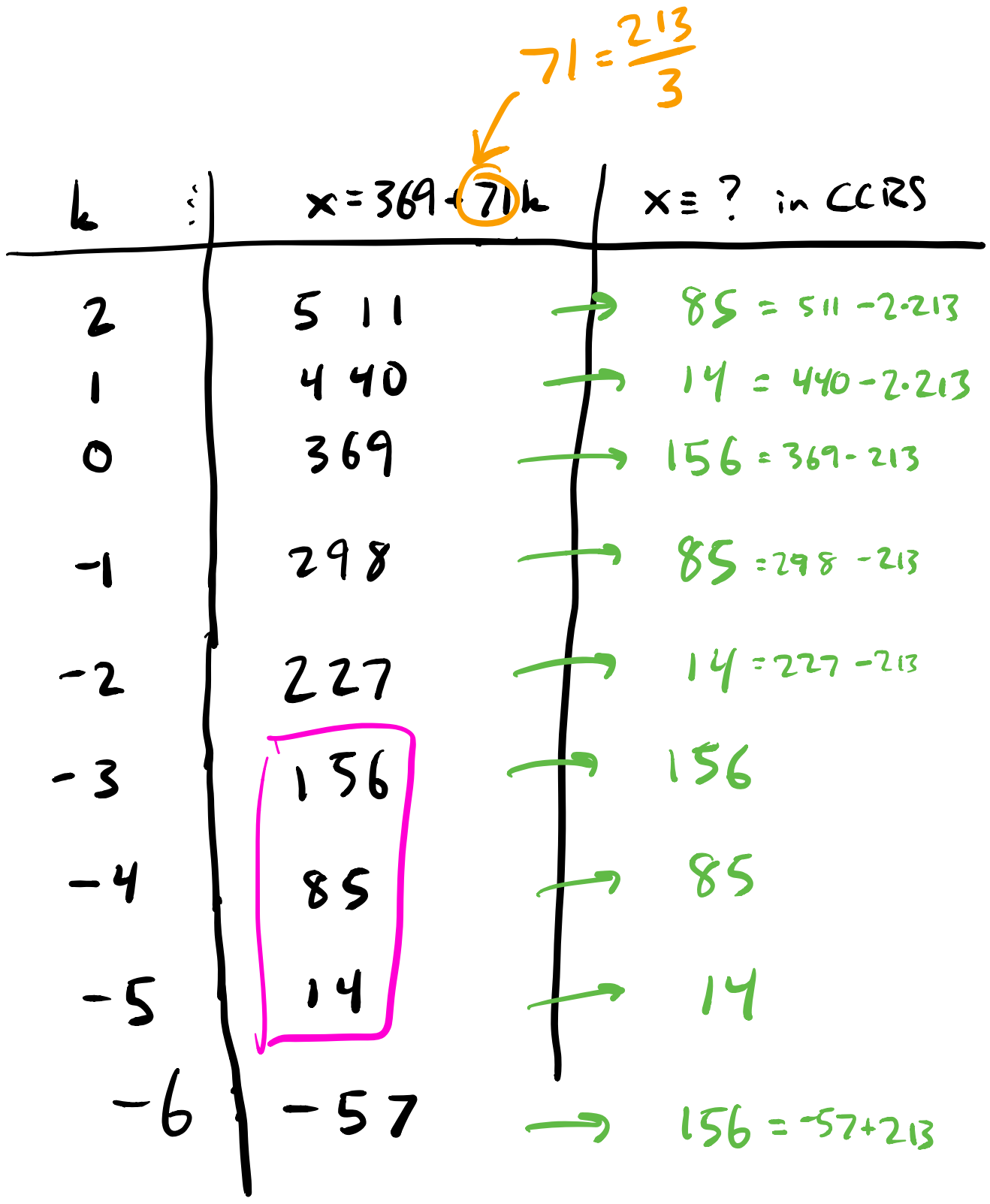
$$y = y_0 - k \cdot \frac{a}{(a,b)}$$

So for this equation, $24x + 213y = 123$

$$x = 369 + k \cdot \frac{213}{3} \qquad y = -41 - k \cdot \frac{24}{3}$$

$$= 369 + 71k \qquad = -41 - 8k$$

Infinitely many solutions for x in the integers.
How many in CCRS?


71 = 213/3
k
x = 369 + 71k
x ≡ ? in CCRS
2
1
0
-1
-2
-3
-4
-5
-6
511
440
369
298
227
156
85
14
-57
85 = 511 - 2·213
14 = 440 - 2·213
156 = 369 - 213
85 = 298 - 213
14 = 227 - 213
156
85
14
156 = -57 + 213

3.24: Use Thm 1.53 to describe all solutions to $ax \equiv b \pmod{n}$ in the CCRS.

- Thm 3.20 $\rightarrow$ Have a solution $\Leftrightarrow (a,n)$ divides b
- ~~If~~ there is one solution, there are (a,n) solutions in the CCRS