1 Write each of the following permutations as a product of disjoint cycles in S_6.

(a) $(1\,2)(2\,3)(3\,4)(4\,5)(5\,6)$

(b) $(3\,4\,5)^{-1}$

(c) $(3\,4\,5)(2\,5\,6\,4)(3\,5\,4)$

(d) $(2\,3)(5\,6)\big((1\,6\,5\,3)(2\,4)\big)(2\,3)(5\,6)$

2 Recall that a **transposition** is a 2-cycle in S_n.

(a) Let $\sigma = (a_1,\ a_2,\ a_3,\ \cdots,\ a_k)$ be a k-cycle. Prove that σ can be written as a product of transpositions. [HINT: Look at problem 1(a).]

(b) Prove that any transposition $(i,\ j)$ can be written as a product of some of the transpositions

$$(1\ 2), (2\ 3), (3\ 4), \ldots, (n-1,\ n).$$

(c) Use parts (a) and (b) to conclude that the transpositions

$$(1\ 2), (2\ 3), (3\ 4), \ldots, (n-1,\ n)$$

generate S_n.