1 Recall that if G is a group and $x \in G$, then the **conjugacy class** containing x is

$$\{gxg^{-1} \mid g \in G\}.$$

Compute all of the conjugacy classes in the following groups.

(a) $\mathbb{Z}_6$

(b) D_5

(c) Q_8

2 Let G be a finite group. Prove that $a^{|G|} = 1$ for every $a \in G$.

3 Let n be a positive integer, and set

$$(\mathbb{Z}_n)^\times = \{a \in \mathbb{Z}_n \mid \gcd(a, n) = 1\}.$$

(a) Prove that $(\mathbb{Z}_n)^\times$ is closed under *multiplication* modulo n.

(b) Prove that $(\mathbb{Z}_n)^\times$ is a group under multiplication modulo n.

(c) Show, by giving an example, that $(\mathbb{Z}_n)^\times$ is not necessarily a cyclic group.

4 Let n be a positive integer, and consider again the group $(\mathbb{Z}_n)^\times$.

(a) Compute the order $|(\mathbb{Z}_n)^\times|$. [HINT: Recall the **Euler ϕ-function** from MA 261.]

(b) Use problem 2 to prove **Euler's theorem:** If n is a positive integer, and a is an integer such that $\gcd(a, n) = 1$, then

$$a^{\phi(n)} \equiv 1 \pmod{n}.$$