

1 Consider the function $\varphi: \mathbb{R}^\times \rightarrow \mathbb{R}^\times$ defined by $\varphi(x) = |x|$.

- (a) Prove that φ is a homomorphism.
- (b) Find the kernel and the image of φ .
- (c) What does the Fundamental Isomorphism Theorem say when applied to φ ?
- (d) Describe the fibers of φ .

2 Prove that $Q_8/\langle -1 \rangle \cong V_4$.

3 Let G be a group, and let $N \trianglelefteq G$ and $H \leq G$. Show that

$$HN = \{hn \mid h \in H, n \in N\}$$

is a subgroup of G .