1 Let G be a group. An isomorphism from G to itself is called an **automorphism** of G.

(a) Let $g \in G$. Prove that the "conjugation by g" map

$$\begin{aligned} \gamma_g \colon G &\to G \\ x &\mapsto gxg^{-1} \end{aligned}$$

is an automorphism.

(b) Prove that

$$\mathrm{Inn}(G) = \{\gamma_g \mid g \in G\}$$

is a group under function composition. This group is called the **inner automorphism group of** G.

2 Let G be a group.

(a) Prove that

$$\begin{aligned} \Gamma\colon G &\to \mathrm{Inn}(G) \\ g &\mapsto \gamma_g \end{aligned}$$

is a surjective homomorphism.

(b) Use the Fundamental Isomorphism Theorem to prove that $G/Z(G) \cong \mathrm{Inn}(G)$.