1 Let R be a ring with identity.

(a) Prove that $(-1)a = -a = a(-1)$ for all $a \in R$.

(b) Prove that $(-1)^2 = 1$ in R.

(c) Prove that if $u \in R$ is a unit, then so is $-u$.

2 Let $S = \{f\colon \mathbb{R} \to \mathbb{R}\}$ be the set of all functions from $\mathbb{R}$ to itself. Addition and multiplication of functions in S are defined *pointwise*, so that if $f, g \in S$, then $f + g$ is the function

$$(f+g)(x) = f(x) + g(x)$$

and fg is the function

$$(fg)(x) = f(x)g(x).$$

(a) Prove that S is a ring under these operations.

(b) Does S have an identity element? Is S commutative?

(c) Let $a \in \mathbb{R}$. Define a function

$$\begin{aligned} \varphi_a\colon S &\to \mathbb{R} \\ f &\mapsto f(a). \end{aligned}$$

That is, φ_a is the **evaluation at** a function. Prove that φ_a is a ring homomorphism.

3 Describe all *ring* homomorphisms $\varphi\colon \mathbb{Z} \to \mathbb{Z}$. [HINT: What can $\varphi(1)$ be?]

4 A ring R is called a **Boolean ring** if $a^2 = a$ for every $a \in R$. Prove that every Boolean ring is commutative.