1 Let $\mathbb{Z}$ be the set of all integers, and let

$$4\mathbb{Z} = \{n \in \mathbb{Z} \mid n = 4k \text{ for some } k \in \mathbb{Z}\}$$

be the set of integers which are multiples of 4.

(a) Let $\sim_4$ be the relation on $\mathbb{Z}$ defined by

$$a \sim_4 b \quad \text{if and only if} \quad a - b \in 4\mathbb{Z}.$$

Prove that $\sim_4$ is an equivalence relation by showing that it is **reflexive**, **symmetric**, and **transitive**.

(b) How many equivalence classes does the relation $\sim_4$ have? Describe them.

2 Let $f\colon X \to Y$ be a surjective map of sets. For each $y \in Y$, the **fiber of** f **over** y is the set

$$f^{-1}(y) = \{x \in X \mid f(x) = y\}.$$

[CAUTION: The fiber $f^{-1}(y)$ is a *subset* of X.]

(a) Consider the surjection $f\colon \mathbb{R} \to \mathbb{R}_{\geq 0}$ given by $f(x) = x^2$. For each $r \in \mathbb{R}_{\geq 0}$, describe the fiber $f^{-1}(r)$.

(b) Let $f\colon X \to Y$ be a surjective map of sets. Prove that the relation

$$x_1 \sim x_2 \quad \text{if and only if} \quad f(x_1) = f(x_2)$$

is an equivalence relation on the set X. Prove that the equivalence classes are the fibers of f.