1 Let $\langle S, *_S \rangle$, $\langle T, *_T \rangle$, and $\langle U, *_U \rangle$ be binary structures.

(a) Recall that the identity map $\mathrm{id}_S \colon S \to S$ is defined by $\mathrm{id}_S(x) = x$ for all $x \in S$. Prove that id_S is an isomorphism. That is, $S \cong S$.

(b) Suppose $\varphi \colon S \to T$ is an isomorphism. Then φ is a bijection, so the inverse map $\varphi^{-1} \colon T \to S$ exists. Prove that φ^{-1} is also an isomorphism. That is, if $S \cong T$, then $T \cong S$.

(c) Suppose $\varphi \colon S \to T$ and $\psi \colon T \to U$ are isomorphisms. Prove that the composition $\psi \circ \varphi \colon S \to U$ is an isomorphism. That is, if $S \cong T$ and $T \cong U$, then $S \cong U$.

2 Let G and H be groups, and let $\varphi\colon G \to H$ be an isomorphism.

(a) Prove that if e_G is the identity element in G, then $\varphi(e_G)$ is the identity element in H.

(b) Let $a \in G$. Prove that $\varphi(a^{-1})$ is the inverse of $\varphi(a)$ in H; that is, in symbols, $\varphi(a^{-1}) = \varphi(a)^{-1}$.