1 Let $\langle R, * \rangle$ be a binary structure with the following properties

- $*$ is associative;
- there exists an identity element $e \in R$ for $*$.

Let $R^\times$ be the set of invertible elements in R,

$$R^\times = \{r \in R \mid \text{there exists } r^{-1} \in R \text{ such that } r * r^{-1} = r^{-1} * r = e\}.$$

Prove that $R^\times$ is closed under $*$ and is a group. ($R^\times$ is called the **group of units** of R.)

2 Let G be a group and let $a \in G$. Prove that, for any $n \in \mathbb{Z}_{>0}$, the inverse element of a^n is $(a^{-1})^n$. That is, $(a^n)^{-1} = (a^{-1})^n$. Your proof should use mathematical induction.

3 Let G be a group with the property that $a^2 = e$ for every $a \in G$. Prove that G is abelian.

4 Let G be a finite group of even order.

(a) Prove that the set
$$t(G) = \{g \in G \mid g \neq g^{-1}\}$$
has an even number of elements.

(b) Use part (a) to prove that there is a non-identity element $a \in G$ such that $a^2 = e$.