1 Let G be a group, and let $H \leq G$ and $K \leq G$ be subgroups.

(a) Prove that $H \cap K \leq G$.

(b) Prove that $H \cup K \leq G$ if and only if either $H \subseteq K$ or $K \subseteq H$.

2 Let G be a group, and let $x, g \in G$.

(a) Prove by induction that $(gxg^{-1})^n = gx^n g^{-1}$.

(b) Prove that $|gxg^{-1}| = |x|$. [HINT: There are two cases: Either $|x| < \infty$ or $|x| = \infty$.]

(c) Deduce that $|ab| = |ba|$ for all $a, b \in G$.

3 Let G be a group with no proper nontrivial subgroups. That is, the only subgroups of G are G itself and $\{e\}$, where $e \in G$ is the identity element. Prove that G is cyclic and that $|G| = p$ for some prime number p.