1 Let G and H be groups, let $\varphi\colon G \to H$ be an isomorphism, and let $a \in G$. Prove that $|\varphi(a)| = |a|$.

2 Let G be a group. The **center** of G is

$$Z(G) = \{z \in G \mid zg = gz \text{ for all } g \in G\}.$$

(a) Prove that $Z(G)$ is a subgroup of G.

(b) Prove that $Z(G)$ is abelian.

(c) Let $n \geq 3$ be an integer. Prove that

$$Z(D_n) = \begin{cases} \{1\} & \text{if } n \text{ is odd;} \\ \{1, r^{n/2}\} & \text{if } n \text{ is even.} \end{cases}$$