1 Recall that

$$D_4 = \langle r, s \mid r^4 = s^2 = 1, rs = sr^{-1} \rangle.$$

(a) Find all cyclic subgroups of D_4.

(b) Find all proper subgroups of D_4 which are not cyclic.

(c) Draw the subgroup lattice of D_4.

2 Recall that

$$Q_8 = \langle -1, i, j, k \mid (-1)^2 = 1,\ i^2 = j^2 = k^2 = ijk = -1 \rangle.$$

As a set, we have

$$Q_8 = \{1, i, j, k, -1, -i, -j, -k\},$$

where $-i$ is shorthand for $(-1) \cdot i$, and similarly for $-j$ and $-k$.

(a) Write the multiplication table for Q_8.

(b) Find all cyclic subgroups of Q_8.

(c) Prove that every proper subgroup of Q_8 is cyclic.

(d) Draw the subgroup lattice of Q_8.

3 Let G be a group and let $g \in G$ be an element. Define a function

$$\begin{aligned} \lambda_g \colon G &\to G \\ x &\mapsto gx. \end{aligned}$$

(a) Prove that λ_g is a bijection from G to itself.

(b) Prove that λ_g is an isomorphism if and only if g is the identity element in G.

4 Let G be a group. Define a relation on G by

$$x \sim y \text{ if and only if there exists } g \in G \text{ such that } y = gxg^{-1}.$$

Prove that $\sim$ is an equivalence relation.