1 Prove that $\mathbb{Q}(\sqrt{5}, \sqrt{7}) = \mathbb{Q}(\sqrt{5} + \sqrt{7})$.

2 The following complex numbers are algebraic over $\mathbb{Q}$. Find the minimal polynomial for each over $\mathbb{Q}$.

(a) $\sqrt{3} + \sqrt{2}$

(b) $\sqrt{3 + i\sqrt{2}}$

(c) $\sqrt{3 + 2\sqrt{2}}$

3 Let E be an extension field of a finite field F, where F has q elements. Let $\alpha \in E$ be algebraic over F of degree n. Prove that $F(\alpha)$ has q^n elements.

4 Let $\alpha = e^{2\pi i/5} = \cos(2\pi/5) + i\sin(2\pi/5) \in \mathbb{C}$.

(a) Find the minimal polynomial of α over $\mathbb{Q}$.

(b) For which positive integer k is $\{1, \alpha, \alpha^2, \ldots, \alpha^k\}$ a basis for $\mathbb{Q}(\alpha)$ as a vector space over $\mathbb{Q}$? Express $\alpha^{-1} = e^{-2\pi i/5}$ as a $\mathbb{Q}$-linear combination of basis elements.

(c) Find the minimal polynomial for $2\cos(2\pi/5)$ over $\mathbb{Q}$. [HINT: Consider $\alpha + \alpha^{-1}$].

(d) Use the polynomial from part (c) to find a simple expression for $\cos(2\pi/5)$.