1 Find a splitting field $E \supseteq \mathbb{Q}$ for each of the following polynomials over $\mathbb{Q}$. In each case, compute the degree $[E : \mathbb{Q}]$.

(a) $x^4 + 1$

(b) $x^4 + 4$

(c) $(x^4 + 1)(x^4 + 4)$

(d) $(x^4 - 1)(x^4 + 4)$

HINT: Note that $\zeta = e^{\pi i/4} \in \mathbb{C}$ is a complex number with the property that $\zeta^2 = i$.

2 The complex numbers $i\sqrt{5}$ and $1+i\sqrt{5}$ are roots of the quartic

$$f(x) = x^4 - 2x^3 + 11x^2 - 10x + 30 \in \mathbb{Q}[x].$$

Let $E \supseteq \mathbb{Q}$ be a splitting field for $f(x)$. Does there exist an automorphism $\sigma \in \mathrm{Aut}(E/\mathbb{Q})$ such that $\sigma(i\sqrt{5}) = 1 + i\sqrt{5}$?

3 Which of the following extensions are normal?

(a) $\mathbb{Q}(i) \supseteq \mathbb{Q}$

(b) $\mathbb{Q}(\sqrt[3]{7}) \supseteq \mathbb{Q}$

(c) $\mathbb{Q}(\omega\sqrt[3]{7}) \supseteq \mathbb{Q}$, where $\omega = e^{2\pi i/3} \in \mathbb{C}$

(d) $\mathbb{Q}(\omega, \sqrt[3]{7}) \supseteq \mathbb{Q}$, where $\omega = e^{2\pi i/3} \in \mathbb{C}$